

## THE POWER OF THE LAST WORD IN LEGISLATIVE POLICY MAKING

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We examine legislative policy making in institutions with two empirically relevant features: agenda setting occurs in real time and the default policy evolves. We demonstrate that these institutions select Condorcet winners when they exist, provided a sufficient number of individuals have opportunities to make proposals. In policy spaces with either pork barrel or pure redistributive politics (where a Condorcet winner does not exist), the last proposer is effectively a dictator or near-dictator under relatively weak conditions.

KEYWORDS: Agenda setting, legislative bargaining, majority rule, dictator, pork barrel politics, distributive politics.

### 1. INTRODUCTION

A CENTRAL OBJECTIVE OF POLITICAL ECONOMY is to provide a comprehensive mapping from institutions and sets of feasible policies into collective choices. Knowledge of this mapping is potentially useful for two reasons. First, it helps us understand why certain institutions tend to deliver particular outcomes and why outcomes differ across institutions. Second, it allows us to evaluate institutions based on likely performance. This enhances our ability to design effective institutions *de novo*, and to identify potentially beneficial changes in rules and procedures for existing institutions. It also helps to explain why certain rules and procedures are common, while others are avoided.

In light of their widespread use, legislative institutions merit particular attention. Although many of these institutions have much in common, specific rules and procedures vary widely. In this paper, we examine majoritarian legislative institutions with two critical features: agenda setting occurs in real time and the default policy evolves.

In institutions with real-time agenda setting, a legislator can take previous proposals and votes into account when making a proposal, conditioning his choice on this information. This contrasts with “advance agenda setting,”

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wherein all of the proposals to be considered are selected before deliberations and voting begin. In practice, most legislative institutions, including the U.S. Congress, provide opportunities for real-time agenda setting.

Evolution of the default policy refers to another aspect of an institution's dynamic structure. The default prevailing at any moment during legislative deliberations is the policy that is implemented if all pending and subsequent motions are defeated. If it is possible to modify the default at most once during deliberations, we say the default is fixed. If it is possible to modify it more than once, we say it evolves. The following two simple examples help to clarify this distinction. First, consider an institution that permits the introduction of one bill, a series of proposals to amend the bill, and a final up-or-down vote against some preexisting default without any possibility of further consideration. Here, the default is fixed: regardless of whether amendments have passed or not, the ultimate outcome is the same if all pending and subsequent motions are defeated. In contrast, consider an institution that permits the introduction of a second bill (pertaining to the same policy issue) after the first is either approved or defeated. Here, the default can evolve: passage of the first bill changes the default prevailing during consideration of the second.

Evolving defaults are particularly relevant when a legislature can examine policies for future periods. For example, tax reform bills commonly phase in changes over several years and sometimes provisions specify expiration dates. The passage (and signing) of such a bill does not prevent the legislature from revisiting the issue at a later date; rather, it changes the default for subsequent deliberations. This possibility is often important in practice.<sup>2</sup>

The literature on legislative bargaining has examined institutions with real-time agenda setting, but fixed defaults (e.g., Baron and Ferejohn (1989), Banks and Duggan (1998, 2000, 2001), Merlo and Wilson (1995), and Diermeier and Merlo (2000)).<sup>3</sup> The literature on sequential agendas has studied institutions with both fixed and evolving defaults, but advanced agenda setting (e.g., Shepsle and Weingast (1984), Ferejohn, Fiorina, and McKelvey (1987), Romer and Rosenthal (1978), Austen-Smith (1987), Banks (1985), Dutta, Jackson, and Le Breton (2001), Banks and Gasmi (1987), and Miller (1980)).<sup>4</sup> The

<sup>2</sup>For example, in 2001, President George W. Bush signed into law new tax legislation that specified changes to be implemented over the course of the following decade. Senate Democratic leader Thomas A. Daschle immediately denounced the measure, promising that "(w)e will revisit these issues. We will try to find ways to make corrections" (Kellser and Eilperin (2001)). Likewise, according to reports in the popular press, "(t)ax analysts warned... that some provisions phase in very slowly, and—if history is a guide—a number of them may never materialize" (Lochhead (2001)). For example, in evaluating the provisions that pertain to the estate tax, one practitioner noted, "(t)he changes have been stretched out for so many years, you can not think that [lawmakers are not] going to come back and revisit this" (Weston (2001)).

<sup>3</sup>In this literature, the default policy is fixed because deliberations terminate and a policy is implemented as soon as it defeats the initial default.

<sup>4</sup>With forward voting, these models have evolving defaults; with backward voting, they have fixed defaults.

remaining permutation—real-time agenda setting with evolving defaults—has received little attention despite its empirical relevance.<sup>5</sup>

This paper represents a step toward understanding legislative institutions with real-time agenda setting and evolving defaults. As such, it does not attempt to model any actual institution in detail. Instead, its object is to identify some of the basic forces at work in these settings by studying a simple institution with the desired features. In the institution we study, legislators are recognized sequentially. Once recognized, a legislator formulates and submits a proposal, which is put to an immediate vote. If a proposal passes, it supersedes the existing default policy and deliberations continue. The policy that emerges from the terminal round of this process is implemented.

Naturally, it is also important to study more realistic and complex institutions with real-time agenda setting and evolving defaults (e.g., by inserting an amendment process during which the default remains frozen). Our analysis of the simple institution is nevertheless useful, both because it is a natural benchmark case against which one can measure the effects of alternative rules and procedures, and because it allows us to study the underlying forces at work in a transparent and tractable setting.

We demonstrate that the simple institution always selects a Condorcet winner when one exists, provided a sufficient number of people have opportunities to make proposals. For the bulk of the paper, we specialize to environments with “pork barrel” policies, which do not have Condorcet winners and which have been widely used in the literature on legislative institutions to study distributive politics. We show that the simple institution vests the final proposer with dictatorial power under relatively weak conditions. Our main result requires complete foreknowledge both of the order in which proposers are recognized and of the number of proposal rounds. However, we show that as long as the group is large, the final proposer’s preferences almost certainly prevail, even with relatively little foreknowledge. We also consider a more flexible variant of the pork barrel policy space, as well as an environment that allows for arbitrary divisions of a fixed prize. In both instances, we find that the combination of real-time agenda setting and evolving defaults tends to concentrate power in the hands of the legislator with the last word. Conceivably, various commonly observed rules and procedures may have come into use because they combat this tendency (a possibility we are pursuing).

Our conclusions are reminiscent of a classic result by McKelvey (1976, 1979), who showed that an agenda setter can achieve his most preferred outcome in a setting where voters consider proposals sequentially but vote myopically, provided the policy space provides sufficient distributional flexibility. For the institutions and policy spaces we consider, a legislator need not control the

<sup>5</sup>The literature on dynamic policy making contains a few models with real-time agenda setting and evolving defaults (Baron (1996), Barron and Herron (1999), Epple and Riordan (1987), and Ingberman (1985)). We discuss their relationship to our work in Section 6.

entire agenda to dictate the outcome; control of the final proposal suffices. Other institutions studied in the literature on legislative bargaining (e.g., Baron and Ferejohn (1989)) and sequential agendas with sophisticated voting (e.g., Shepsle and Weingast (1984) and Ferejohn, Fiorina, and McKelvey (1987)) do not tend to produce dictatorial or near-dictatorial outcomes, and in many cases generate majoritarian and even “universalistic” policies.

The paper is organized as follows. Section 2 describes the model and derives some general properties. Section 3 characterizes outcomes with pork barrel policies. Section 4 studies the role of predictability (that is, foreknowledge concerning the order of proposers and the number of proposal rounds). Section 5 examines outcomes with other policy spaces. Section 6 describes an extension to dynamic settings. Section 7 concludes. Some of the proofs appear in the text; others appear in the [Appendix](#).

## 2. THE MODEL

Consider a collective decision making body (for example, a legislature) that consists of  $N$  individuals, labeled  $l = 1, \dots, N$ , where  $N \geq 3$ . To avoid complications that arise from tie votes, we assume that  $N$  is odd. Let  $M \equiv (N + 1)/2$  denote the size of the smallest majority. The group must select a policy  $p \in P$ , where  $P$  denotes the set of feasible policies. We use  $v_l(p)$  to denote the payoff for individual  $l$  when the group selects policy  $p$ . With the exception of Section 5.2, we impose the following assumption throughout:

**ASSUMPTION A1:** *Set  $P$  is a finite set and every individual has strict preferences over its elements:  $p \neq p' \Rightarrow v_l(p) \neq v_l(p')$ .*

With finiteness, the nonindifference requirement is generic and, therefore, relatively innocuous.

### 2.1. Institutions

We study a class of simple institutions in which the group selects a policy over the course of  $T$  rounds (with  $T$  finite).<sup>6</sup> Activity prior to each round  $t$  establishes a default policy  $p_{t-1}$ . The initial default policy  $p_0$  is given exogenously. Round  $t$  begins when some individual, denoted  $i(t)$ , is recognized and given the opportunity to make a proposal  $p_t^m$ , which can be any element of  $P$ . The proposal is then put to an immediate vote against  $p_{t-1}$ . If it receives majority approval (“passes”), it displaces  $p_{t-1}$  as the default policy ( $p_t = p_t^m$ ). If

<sup>6</sup>Finiteness of  $T$  is particularly appropriate in a setting where one thinks of policies as time-dated actions: one cannot meaningfully deliberate the policy to prevail on January 1, 2005 once that date has passed. For an elaboration of this perspective, see the discussion of dynamic policies in Section 6.

it does not pass, the default policy remains the same for the following round ( $p_t = p_{t-1}$ ). The policy that emerges from the last proposal round,  $p_T$ , is implemented.

We assume that individuals can condition their actions—proposals as well as votes—on prior actions, all of which are observable. Thus, in addition to an evolving default policy, the institutions considered here also incorporate real-time agenda setting.

With the exception of Section 4, we assume throughout that both the number of rounds and the identity of each round’s proposer are fixed and known to all in advance. We use  $J$  to denote the set of individuals who are recognized at least once.

### 2.2. Behavioral Assumptions

We study pure strategy subgame perfect equilibria with the following property: in each period  $t$ , if the continuation outcome depends only on  $p_t$ , individuals vote as if they are pivotal.<sup>7</sup> Each individual compares the continuation outcome if a proposal passes with the continuation outcome if it is defeated, and casts his vote for the option with the preferred continuation outcome. We assume “as-if-pivotal” voting to deal with the familiar problem of indifference among nonpivotal voters, which otherwise gives rise to a vast multiplicity of unreasonable equilibria wherein people vote contrary to their true preferences. When used in the following sections, the term “equilibrium” subsumes these restrictions.

### 2.3. Some General Properties

Under Assumption A1, the equilibrium outcome for each subgame is unique, even though (as we will see) the equilibrium strategy profile often is not. For all  $t$ , let  $f_t(p) \in P$  denote the unique final policy, or outcome, eventually enacted when  $p$  is the default at the beginning of round  $t$ . We refer to  $f_t(p)$  as the *continuation outcome*. Let  $F_t \equiv \bigcup_{p \in P} f_t(p)$  denote the set of all possible continuation outcomes in round  $t$ . Clearly,  $F_t \subseteq F_{t+1}$ , with  $F_{T+1} \equiv P$ .

Consider  $i(t)$ ’s choice of a proposal in round  $t < T$ . If he proposes  $p_t^m$  and it passes, the eventual outcome is  $f_{t+1}(p_t^m)$ . If his proposal does not pass, the eventual outcome is  $f_{t+1}(p_{t-1})$ . It is helpful to think of  $i(t)$  as proposing the continuation outcome  $f_{t+1}(p_t^m)$  (rather than the policy  $p_t^m$ ), which voters compare with the alternative continuation outcome  $f_{t+1}(p_{t-1})$  (rather than with the policy  $p_{t-1}$ ). When selecting a proposal,  $i(t)$  can either accept the outcome  $f_{t+1}(p_{t-1})$  (e.g., by proposing  $p_{t-1}$ ) or change the outcome to any element of  $F_{t+1}$  that at least  $M$  individuals prefer to  $f_{t+1}(p_{t-1})$ . Thus,  $i(t)$  in effect

<sup>7</sup>If the continuation outcome depends on period  $t$  voting as well as on  $p_t$ , an individual’s preferred option may depend on how others vote, in which case the meaning of as-if-pivotal voting is ambiguous. However, as will become clear below, this never occurs under Assumption A1.

chooses the ultimate outcome by solving  $\max_p v_{i(t)}(p)$  subject to two restrictions: (i)  $p \in F_{t+1}$  and (ii) either  $p = f_{t+1}(p_{t-1})$  or at least  $M$  individuals prefer  $p$  to  $f_{t+1}(p_{t-1})$ . Using  $p'$  to denote the solution to this problem, individual  $i(t)$  proposes any  $p_i^m$  such that  $f_{t+1}(p_i^m) = p'$ . Note that the winning coalition votes for  $p_i^m$  not because they prefer it to the current default  $p_{t-1}$ , but because they prefer the eventual outcome  $p'$  to  $f_{t+1}(p_{t-1})$ . When  $F_{t+1}$  is a proper subset of  $P$ , there may be more than one proposal  $p_i^m$  that satisfies  $f_{t+1}(p_i^m) = p'$ , which is why equilibrium strategies are not necessarily unique.

Notice that the institution described above—one with  $T$  proposal rounds, recognition order  $i(t)$  (for  $t = 1, \dots, T$ ), policy space  $P$ , and initial default  $p_0$ —produces exactly the same outcome as one with  $T - 1$  proposal rounds, recognition order  $i(t)$  (for  $t = 1, \dots, T - 1$ ), policy space  $F_T$ , and initial default  $f_T(p_0)$ . This follows immediately from the fact that, in round  $T - 1$  of the first institution, everyone understands that a vote for policy  $p$  is actually a vote for the continuation outcome  $f_T(p)$ . Accordingly, if we delete the final round while appropriately reducing the policy set and switching the initial default policy, the final outcome is unaffected. One can solve for the final outcome by doing this repeatedly.

Our first result identifies conditions under which the institution selects a Condorcet winner, provided one exists. Clearly, a Condorcet winner need not always prevail (e.g., if there is a single proposal round and the proposer prefers the initial default policy to the Condorcet winner). However, it does emerge as the final outcome whenever a sufficient number of individuals can make proposals, or when the set of proposers includes someone who prefers it to all other alternatives. Formally:

**THEOREM 1:** *Suppose Assumption A1 holds and  $P$  contains a Condorcet winner  $p^c$ . Then  $p^c$  is the final outcome regardless of the initial default policy whenever either of the following statements holds:*

- (i) *At least  $M$  individuals can make proposals ( $|J| \geq M$ ).*
- (ii) *The most preferred policy of some proposer is  $p^c$ .*

**PROOF:** First we claim that once the Condorcet winner  $p^c$  is either the default policy or a proposal in any round, it must be the final outcome. We show this by induction on the number of rounds remaining. It is obviously true when there is one round left. Suppose it is true with  $s - 1$  rounds left and consider the case with  $s$  rounds left. Imagine that  $p^c$  is either the default policy or the proposal and that, contrary to our claim, the final outcome is  $p' \neq p^c$ . If  $p^c$  wins in the current round, it becomes the default policy in the next round, which means it also becomes the final outcome. Whereas a majority prefer  $p^c$  to  $p'$ ,  $p^c$  must win in the current round, so we have a contradiction.

Now suppose, contrary to the theorem, that either condition (i) or condition (ii) is satisfied and that the outcome is some  $p' \neq p^c$ . Under either condition, at least one of the proposers strictly prefers  $p^c$  to  $p'$ . Because, by the first

step, any such proposer can change the final outcome to  $p^c$  by proposing it, we have a contradiction. *Q.E.D.*

From Theorem 1, it follows that the initial default policy is irrelevant provided there are enough proposal rounds and sufficient diversity of proposers:

**COROLLARY 1:** *Suppose Assumption A1 holds,  $T > |P| - 3$ , and at least  $M$  different individuals can make proposals prior to round  $t^* \equiv T - |P| + 3$ . Then the equilibrium outcome is independent of the initial default  $p_0$ .*

**PROOF:** We begin with two observations. First, if the continuation set  $F_{t+1}$  contains a Condorcet winner, so will  $F_t$ . Second, if  $F_{t+1}$  does not contain a Condorcet winner,  $F_t$  must be strictly smaller than  $F_{t+1}$  (notice that whenever  $i(t)$ 's least favorite policy in  $F_{t+1}$  is not a Condorcet winner,  $i(t)$  can always propose an alternative policy that defeats it). Applying these two observations recursively,  $F_{t+1}$  either contains a Condorcet winner or contains at most  $|P| - (T - t)$  different policies. It follows that  $F_{t^*+1}$  either contains a Condorcet winner or contains exactly three policies (in which case  $F_{t^*}$  contains two). In either case,  $F_{t^*}$  must contain a Condorcet winner. Corollary 1 follows from applying Theorem 1 to the transformed (and equivalent) game with policy space  $F_{t^*}$  and only the first  $t^* - 1$  proposal rounds. *Q.E.D.*

Corollary 1 is of interest because it establishes that sufficiently early deliberations and haggling over the initial default policy are inconsequential in this institution. We discuss an application of this property in Section 6.

### 3. OUTCOMES WITH PORK BARREL POLICIES

In most of this paper we assume that policies consist of combinations of projects, each of which benefits a single member of the group at a cost to all others. These “pork barrel” policies provide a natural starting point for our analysis, both because they capture important features of many collective decision problems and because they feature prominently in the previous literature on legislative politics (see, e.g., Ferejohn (1974) or Ferejohn, Fiorina, and McKelvey (1987)). We consider other policy spaces in Section 5.

#### 3.1. The Pork Barrel Policy Space

Each individual is associated with a single project. Let  $E \equiv \{1, \dots, N\}$  denote the set of all projects. Each  $l \in E$  produces highly concentrated benefits and diffuse costs. In particular, project  $l$  generates a benefit  $b_l > 0$  for individual  $l$  and a cost  $c_l > 0$  for everyone (including  $l$ ). A policy  $p$  consists of a list of projects. Payoffs are additively separable:

$$v_l(p) = - \sum_{j \in p} c_j + \begin{cases} b_l, & \text{if } l \in p, \\ 0, & \text{otherwise.} \end{cases}$$



The set of feasible policies  $P$  is the set of all possible combinations of projects (that is, subsets of  $E$ ). The set  $P$  is finite and includes the empty set  $\emptyset$ , which represents inaction (nothing is implemented). We refer to this as a *pork barrel policy space*.

We impose two additional assumptions:

**ASSUMPTION A2:** *Each project is less costly than any combination of  $M - 1$  projects:  $|p| \geq M - 1$  implies that, for all  $j$ , we have  $c_j < \sum_{j' \in p} c_{j'}$ .*

**ASSUMPTION A3:** *A mutually beneficial policy (relative to  $p = \emptyset$ ) exists for all coalitions that consist of  $M$  or fewer individuals. In particular, for every policy  $p$  with  $|p| \leq M$ ,  $b_l > \sum_{j \in p} c_j$  for all  $l \in p$ .*

Assumption **A2** restricts the degree to which costs can vary across projects. It rules out the case of  $N = 3$  (because then  $M - 1 = 1$ ), but is easily satisfied when the group is large. Assumption **A3** guarantees the existence of policies that are preferred to inaction by every bare-majority coalition. To understand the role of Assumption **A3**, note that when no mutually beneficial policy (relative to inaction) exists for any bare-majority coalition,  $p = \emptyset$  is a Condorcet winner.

This policy space gives rise to rich distributive politics and has no Condorcet winner. Efficiency is at stake in some instances, but not in others. In fact, for some parameterizations, the entire set is Pareto unranked. Notably, the universalistic policy  $p = E$  need not maximize aggregate surplus and may even be Pareto inefficient.

### 3.2. Equilibrium Outcomes

Our next result demonstrates that with pork barrel policies, the institution described in Section 2.1 can effectively endow the final proposer with dictatorial power. Ironically, this occurs when the proposal process is sufficiently *inclusive*, irrespective of the initial default policy, the order of recognition, or the costs and benefits associated with any particular project (except for those imposed by Assumptions **A2** and **A3**).<sup>8</sup>

**THEOREM 2:** *Consider a pork barrel policy space and suppose Assumptions **A1**–**A3** hold. If either a sufficient number of individuals can make proposals*

<sup>8</sup>The result allows for the possibility that any particular individual may be recognized once, more than once, or not at all. Any reordering of proposers prior to the final round is irrelevant and the same individual can have the opportunity to make proposals in consecutive rounds. It is natural to conjecture that consecutive proposals are redundant, but this is not always the case. For example, with  $T = 1$  and  $p_0 = \emptyset$ , the outcome is a policy with  $M$  projects including  $i(T)$ . However, with  $T = 2$ ,  $p_0 = \emptyset$ , and  $i(1) = i(2)$ , the outcome consists of  $i(2)$ 's project and nothing else (this follows from Theorem 2).



( $|J| > M$ ) or  $i(T)$  proposes more than once, the unique outcome is the policy that consists of  $i(T)$ 's project and nothing else (namely, the policy  $\{i(T)\}$ ).

PROOF: We prove the theorem by establishing three claims.

CLAIM 2.1: *If the default policy for round  $T$  includes only  $i(T)$ 's project, then it is the outcome.* Agent  $i(T)$  can achieve this outcome simply by proposing the default policy; because this is his global optimum, it is his best achievable outcome.

CLAIM 2.2: *Regardless of the default policy for round  $T$ , the outcome never includes more than  $M$  projects.* Suppose on the contrary that it contains  $M + m$  projects, with  $m > 0$ . For the policy to pass, the votes of at least  $m$  of the agents whose projects are included in the outcome are not needed. Delete the projects of those  $m$  inessential voters from the proposal. A majority (including  $i(T)$ ) are now even better off, so this policy passes as well. Thus, the original proposal was not a best response for  $i(T)$ .

CLAIM 2.3: *Regardless of the default policy for round  $T$ , the outcome always includes  $i(T)$ 's project.* Here there are several cases to consider. For each case, we prove that  $i(T)$  can make a successful proposal that includes his own project and that yields  $i(T)$  strictly positive payoffs. Although this may not be a best response, it is strictly better than anything that does not include  $i(T)$ 's project. Thus,  $i(T)$ 's best response must also include  $i(T)$ 's project.

CASE 1: Agent  $i(T)$ 's project is included in the default policy. If  $M$  or fewer projects are included,  $i(T)$  can propose the default policy. If  $M + m$  projects are included with  $m > 0$ ,  $i(T)$  can drop  $m$  projects. This passes and produces a strictly positive payoff for  $i(T)$ .

CASE 2: Agent  $i(T)$ 's project is not included and the default policy has fewer than  $M$  components. Then  $i(T)$  can propose a policy with  $M$  components (including his own), such that none of these components is in the default policy. This passes and produces a strictly positive payoff for  $i(T)$ .

CASE 3: Agent  $i(T)$ 's project is not included and the default policy has at least  $M$  components. Then  $i(T)$  can make a proposal dropping  $M - 1$  elements and adding his own. By Assumption A2, this passes and produces a strictly positive payoff for  $i(T)$  (because it has no more than  $M$  components).

Claims 2.1 and 2.3 imply that the final outcome always includes  $i(T)$  and at most  $M - 1$  other projects. Of the set of possible outcomes that could follow from all period  $T$  default policies,  $\{i(T)\}$  (which belongs to this set by Claim 2.1) is a Condorcet winner. Applying Theorem 1 completes the proof,

provided either that  $i(T)$  is recognized twice or that at least  $M$  distinct individuals are recognized in rounds 1 through  $T - 1$ , which is always the case when  $|J| > M$ . *Q.E.D.*

The intuition for this result is straightforward. Given the final proposer's optimal strategy, the outcome for the last round always includes his project and excludes the projects of  $M - 1$  other individuals (whose votes are not needed to establish a majority). In previous rounds, the individuals who expect to be excluded will join with the last proposer in trying to reduce costs by eliminating other projects. This group can prevail because it constitutes a majority.<sup>9</sup>

We emphasize the perversity of this outcome. When, for example, the initial default policy is the null policy  $\emptyset$ , all individuals other than  $i(T)$  *strictly prefer* it to the final outcome. If the group simply failed to meet, everyone would be better off except  $i(T)$ . Thus, the institution induces the group to choose a policy that is contrary to the interests of almost every member, even though no proposal can pass without majority support.

The result also has the following counterintuitive implication: with this policy space, reforms that appear to be inclusive from a procedural perspective (by guaranteeing more legislators the right to make a proposal) can have the unintended effect of concentrating political power. To see this, suppose that  $p_0 = \emptyset$ , and compare outcomes when  $T = 1$  and when both  $T$  and  $|J|$  are greater than  $M$ . In the first instance, the outcome includes the project  $i(T)$  and the  $M - 1$  least costly projects other than  $i(T)$ . In the second instance, only  $i(T)$  is implemented. Thus,  $i(T)$  would prefer to give others opportunities to make proposals, and others would collectively prefer to avoid these opportunities.

In proving the result, we relied on Assumption A2, which requires  $N \geq 5$  (see Case 3 in the argument for Claim 2.3). For  $N = 3$ , it is easy to construct counterexamples.<sup>10</sup>

For groups with large numbers of members, Theorem 2 imposes both a strong institutional requirement (more than  $M$  members can make proposals) and a strong behavioral requirement (members understand the equilibrium of a game with many rounds). This raises a natural question: Does the last proposer have significantly less power when there are few rounds and only a small fraction of members can make proposals? When fewer than  $M + 1$  individuals can make proposals, there are *always* combinations of recognition orders and initial default policies for which  $\{i(T)\}$  is not the outcome. In this sense,

<sup>9</sup>Though we have assumed, in defining pork barrel policy spaces, that the cost of a given project is the same for each individual, a careful reading of the preceding proof reveals that this is inessential. When each project's cost can differ across individuals, the result holds provided Assumptions A1 and A2 are satisfied for each individual.

<sup>10</sup>Suppose  $N = 3$  and  $i(T)$  is associated with the highest cost project. If the default for round  $T$  consists of the other two projects, the outcome will be the lowest cost project, and  $i(T)$ 's project will be excluded.

we cannot relax the requirement that  $|J| > M$ . However, even with small  $T$  (and hence small  $|J|$ ), nondictatorial outcomes are unusual in the sense that a large fraction of recognition orders lead to the dictatorial outcome  $\{i(T)\}$ . The following result, proven in the [Appendix](#), provides a lower bound for this fraction. We focus here on recognition orders where no individual is recognized twice in a row.<sup>11</sup> Let  $Q^*$  be the smallest number  $Q$  such that every project is less costly than every collection of  $Q$  projects. Obviously,  $Q^* \geq 2$  and, from Assumption [A2](#) we know that  $Q^* \leq M - 1$ .

**THEOREM 3:** *Consider a pork barrel policy space that satisfies Assumptions [A1](#)–[A3](#) and suppose no player is recognized twice in a row. The fraction of recognition orders that generate the outcome  $\{i(T)\}$  for all possible initial default policies  $p_0$  is not less than*

$$B(N, T) \equiv \begin{cases} \frac{1}{2} - \frac{Q^*-1}{N}, & \text{for } T = 2, \\ 1 - \frac{Q^*-1}{N} \left(\frac{M-2}{N-1}\right)^{T-2}, & \text{for } T \geq 3. \end{cases}$$

Assuming the recognition order is selected at random before the game begins and all orders are equally likely, we can interpret the function  $B(N, T)$  as a lower bound for the probability that  $\{i(T)\}$  is the outcome, regardless of the initial status quo. When  $T \geq 3$ , this bound exceeds  $1 - \frac{Q^*-1}{N} \left(\frac{1}{2}\right)^{T-2}$ , which is always greater than 0.75, and quickly converges to unity as either  $T$  or  $N$  increases (with  $Q^*$  constant). Although [Theorem 2](#) may create the impression that dictatorial outcomes are unlikely in very large groups (because it is difficult to achieve  $|J| > M$ ), [Theorem 3](#) demonstrates that this is not the case. On the contrary, with  $N$  large, the last proposer is almost certain to dictate the outcome, even when  $T$  is small (which necessarily implies that  $|J|$  is much less than  $M$ ).

So far, we have assumed that the number of rounds and the recognition order are known to all at the outset, and we have focused our attention on a particular policy space. In the next two sections, we investigate the robustness of our conclusions with respect to these two assumptions.

#### 4. THE ROLE OF PREDICTABILITY

Here we examine the role of predictability while continuing to focus on pork barrel policy spaces. Our previous assumptions concerning predictability—that the number of rounds and the recognition order are known in advance—represent one extreme. At the opposite extreme, one could assume that the identity of the proposer is not revealed until the outset of each round and that the game terminates without warning. It is easy to construct examples for

<sup>11</sup>One can prove a similar result for the case where individuals can be recognized twice in a row.

which the dictatorial power of the last proposer does *not* survive in this alternative setting. Between these two extremes, however, there is a spectrum of intermediate cases with varying degrees of advance notification concerning the identities of proposers and the timing of termination. In practice, some degree of predictability may arise for a variety of reasons. Upcoming proposers may be announced in advance or their selection may be delegated to a predictable agent (e.g., a chair). Likewise, for time-sensitive decisions, the number of remaining opportunities for making proposals may be apparent when the deadline is near.

Formally, we assume that group members learn the identity of the round  $t$  proposer at the outset of period  $t - K_1$ , and learn the value of  $T$  at the outset of period  $T - K_2$ . The case of  $K_1 = K_2 = T - 1$  corresponds to full predictability, while the case of  $K_1 = 0$  and  $K_2 = -1$  corresponds to no advance notification.<sup>12</sup>

Theorem 3 implies that even a relatively small amount of advance notification endows the last proposer with substantial power. In particular, suppose that  $K_1, K_2 \geq 2$ , so that the number of rounds and recognition order are completely revealed by the start of round  $T - 2$ . Assuming all recognition orders are equally likely (with no one proposing twice in a row),  $B(N, 3) > 1 - \frac{Q^* - 1}{2N}$  provides a lower bound on the probability of implementing  $\{i(T)\}$ . Notice that this bound converges to unity as  $N$  approaches infinity. Accordingly, if the group is sufficiently large, the preferences of the last proposer almost certainly dictate the outcome.

Our next theorem (proven in the [Appendix](#)) shows that, provided  $K_2 \geq 2$ , approximate dictatorship holds in large groups even if legislators can only predict the next proposer (that is, with  $K_1 = 1$ , the smallest possible degree of foreknowledge).<sup>13</sup>

**THEOREM 4:** *Consider a pork barrel policy space that satisfies Assumptions A1–A3. Suppose  $T$  and  $i(t)$  (for all  $t \leq T$ ) are selected randomly in such a way that (i) every member has the same probability of serving as the proposer in each period and (ii) no member proposes in two consecutive rounds. In addition, suppose group members receive notification concerning the identity of proposers at least one period in advance (i.e.,  $K_1 \geq 1$ ) and concerning the timing of termination no later than at the outset of round  $T - 2$  (i.e.,  $K_2 \geq 2$ ). Then the outcome  $\{i(T)\}$  occurs with probability no lower than  $1 - \frac{1}{N} - \frac{Q^* - 2}{N - 1}$ .*

<sup>12</sup>When  $K_2 = 0$ , group members learn that they are in the last round at the outset of round  $T$ , which means there is some advance notification. When  $K_2 = -1$ , group members do not learn that they are in the last round until it has ended, which means there is no advance notification.

<sup>13</sup>As with Theorem 3, one can prove a qualitatively similar result that allows for the possibility that individuals make proposals in consecutive rounds. There are also other ways to extend the result on approximate dictatorship in large groups. For example, one can show that it holds when the members are recognized in random order with no advance notification, with each member receiving exactly one opportunity to make a proposal. (In that case,  $T$  is known at the outset and the identities of the last two proposers—but not their order—are known at the start of round  $T - 2$ .)

What can we say about the case where  $K_1 = K_2 = 1$ ? From Theorem 3, we know that  $B(N, 2) = \frac{1}{2} - \frac{Q^* - 1}{N}$  provides a lower bound on the probability of implementing  $\{i(T)\}$  for this case (assuming all recognition orders are equally likely with no one proposing twice in a row). This bound converges to  $1/2$  as  $N$  goes to infinity. Although this finding implies that the last proposer often dictates the outcome in large groups, it does not rule out the possibility that he fails to dictate the outcome with substantial likelihood.

The analysis of this section raises the possibility that, for institutions with real-time agenda setting and evolving default policies, it may be possible to reduce the power of the last proposer by creating a high level of procedural uncertainty. We leave this conjecture for future research.

## 5. THE ROLE OF THE POLICY SPACE

From Theorem 1, it is immediately apparent that our central result (Theorem 2) is sensitive to the structure of the policy space. For example, in the standard Downsian setting with a unidimensional policy space and single-peaked preferences, we know that the median “bliss point” is a Condorcet winner. Theorem 1 tells us that as long as a sufficient number of people have opportunities to make proposals, the institution studied here will select this median bliss point even if it differs radically from the policy most preferred by the last proposer.

A critical difference between the pork barrel policy space and the Downsian setting is that the former provides fairly rich opportunities for redistributing payoffs, whereas the latter does not. It is important to determine whether our result is special to the pork barrel setting or is robust within the set of environments that provide substantial scope for redistribution. In this section, we consider two alternative policy spaces. The first generalizes the pork barrel setting by allowing legislators to benefit from multiple projects. The second provides complete distributional flexibility (and has been widely used in political-economy models). Throughout, we return to the case in which the order of recognition and number of rounds are known by all in advance. In each case, we find that, under relatively weak conditions, the final proposer has dictatorial or near-dictatorial power. In a previous version of this paper, we showed that our central result also extends, under appropriate assumptions, to settings in which subgroups of legislators have common interests.

### 5.1. *Augmented Pork Barrel Politics*

The assumption that each legislator seeks to implement one and only one project is restrictive. In this section, we allow for the possibility that a legislator may benefit from a variety of distinct pork barrel projects. This adds richness and distributional flexibility to the policy space. Under relatively weak conditions, we obtain a generalization of Theorem 2: the legislature implements *all* of the projects that benefit the final proposer and nothing else.

Imagine that each individual  $l$  is associated with  $R_l \geq 1$  projects. Project  $(l, r)$  (that is, the  $r$ th project associated with the  $l$ th individual) imposes an identical cost  $c_{lr}$  on everyone, generates a benefit  $b_{lr}$  for legislator  $l$ , and benefits no one else.<sup>14</sup> As before, payoffs are additive across policies. The policy set  $P$  is once again the set of all possible combinations of projects. We call this an *augmented pork barrel* policy space.

To state our assumptions, we require some additional notation:  $\bar{R}$  denotes the largest number of projects associated with any one individual,  $\bar{c}$  denotes the highest (and  $\underline{c}$  the lowest) cost associated with any project, and  $\underline{b}$  denotes the lowest benefit associated with any project. We continue to impose Assumption A1, but replace Assumptions A2 and A3 with the following:

ASSUMPTION A4: *We have  $\bar{R}\bar{c} < (M - 1)\underline{c}$ .*

ASSUMPTION A5: *We have  $\underline{b} > \bar{c}(M - 1 + \bar{R})$ .*

Assumption A4 imposes an upper bound on the total costs of the projects associated with any individual.<sup>15</sup> Because there is no reason to think that the number of projects that benefit a particular individual would grow with the size of the legislature, this is not a demanding requirement when the legislature is large. Assumption A5 implies that, for any minimal majority, any policy that consists of exactly one project for all but one member and all projects for the remaining member is mutually beneficial.<sup>16</sup>

**THEOREM 5:** *Consider an augmented pork barrel policy space that satisfies Assumptions A1, A4, and A5. Provided that either  $|J| > M$  or  $i(T)$  proposes more than once, the unique outcome is the policy consisting of all projects associated with  $i(T)$  and nothing else.*

Theorem 5 generalizes Theorem 2 (subject to the qualifications noted in footnote 15), but the proof is considerably more involved. The structure of the argument and the core intuition are essentially unchanged: We argue that the

<sup>14</sup>For the proof of Theorem 5, we actually only use the fact that the ranking of the policy set by costs is the same for all individuals. Thus, the assumption of identical costs can be weakened somewhat.

<sup>15</sup>For the case of  $R_i = 1$  for all  $i$ , Assumption A4 is somewhat more restrictive than Assumption A2 because it is written in terms of bounds rather than in terms of the costs of particular collections of projects. One can state another version of Assumption A4 in terms of collections of projects and thereby obtain a true generalization of Assumption A2, but the notation is less compact. A similar comment applies to Assumption A5 in relation to Assumption A3.

<sup>16</sup>Assuming that the total cost of a project does not depend on the size of the legislature, this requirement is also less demanding with  $N$  large. Let  $\bar{C}$  denote the total cost of the most expensive project and assume that  $\bar{c} = \bar{C}/N$ . Then Assumption A5 requires  $\underline{b} > \bar{C}(\frac{1}{2} + \frac{1}{2N} + \frac{\bar{R}-1}{N})$ . Notice that the right-hand side declines monotonically with  $N$ .

dictatorial outcome defeats all other policies in the set of final-round continuation outcomes  $F_T$  and then we apply Theorem 1. By definition,  $i(T)$  prefers the dictatorial outcome to all others. As before, each policy  $p \in F_T$  involves no projects for at least  $M - 1$  individuals other than  $i(T)$ , who therefore prefer every less costly continuation outcome to  $p$ . Accordingly, if the dictatorial outcome is the least costly policy in  $F_T$ , it majority-defeats all other continuation outcomes, as required. To demonstrate that it is indeed the least costly continuation outcome, one must rule out the existence of less expensive continuation outcomes, including some, but not all, of  $i(T)$ 's projects (a step which has no counterpart in the proof of Theorem 2).

### 5.2. Pure Distributive Politics

Finally, we examine the canonical problem of dividing a fixed prize; that is, we assume that the policy space is the unit simplex. This introduces unrestricted distributional possibilities while completely eliminating considerations of efficiency.<sup>17</sup>

We demonstrate that as long as there are at least three individuals and three proposers, there is always an equilibrium in which the last proposer obtains the entire prize. However, there are also equilibria in which he obtains as little as half the prize. To resolve this multiplicity, we propose a simple refinement and show that it selects outcomes for which the last proposer obtains nearly the entire prize in large groups. These results are at once stronger and weaker than Theorem 2. They are stronger in the sense that they require only  $N \geq 3$  and  $|J| \geq 3$  (or simply  $T \geq 2$  in the case of the refinement), rather than  $N \geq 5$  and  $|J| > M$ . They are weaker in the sense that dictatorship is not the only possible equilibrium outcome and, in that, our refinement points to near-dictatorship in large groups, rather than full dictatorship in all cases.

The environment considered here violates Assumption A1 (finiteness and nonindifference). This complicates the definition of as-if-pivotal voting, because the same default may lead to different outcomes, depending on the composition of the vote. We study pure strategy subgame perfect equilibria with the following property: In each period  $t$ , individuals always vote for a policy when it *unambiguously* yields a preferred continuation outcome relative to the alternative (that is, regardless of the composition of votes). This is a relatively weak requirement, because it imposes no restriction on voting when one alternative leads to a better continuation outcome for some voting profiles, but not for others.

<sup>17</sup>Efficiency is not an issue in any case because all outcomes in  $F_T$  are necessarily weakly Pareto efficient, irrespective of the policy space. One can therefore think of the unit simplex as representing the efficient frontier of the utility possibility space in any environment with unrestricted distributional possibilities and an outside option (establishing a minimum payoff) for each member.



We begin with a result that characterizes the entire set of equilibrium outcomes.<sup>18</sup>

**THEOREM 6:** *Suppose the set of feasible payoffs is the unit simplex and  $N \geq 3$ . Then the last proposer receives a payoff of at least  $\frac{1}{2}$ . In addition, suppose  $|J| \geq 3$  and the last two proposers differ. Then, for any  $p_0$  and  $\alpha \in [\frac{1}{2}, 1]$ , there exists an equilibrium in which  $i(T)$  receives  $\alpha$ .*

It is possible to sustain the wide range of outcomes described in Theorem 6 because indifferent voters are often collectively pivotal and one is free to prescribe fortuitous choices for them. This observation raises the concern that many of the associated equilibria are fragile. To explore this possibility, we consider a modified version of the game in which each individual votes for the prevailing default unless the current proposal improves his payoff by at least  $\varepsilon \neq 0$ . With  $\varepsilon > 0$ , proposers must overcome a small bias in favor of no change (players vote against proposals when indifferent); with  $\varepsilon < 0$ , proposers benefit from a small bias in favor of change (players vote for proposals when indifferent). The purpose here is to impose a consistent criterion for resolving indifference and thereby rule out equilibria that rely on fortuitous patterns, with individuals voting both for and against proposals when indifferent, depending on circumstances. We say that an equilibrium outcome  $p$  is robust if it is the limit of equilibrium outcomes for some sequence of modified games with  $\varepsilon$  converging to zero.

**THEOREM 7:** *Suppose the set of feasible payoffs is the unit simplex,  $N \geq 3$ , and  $T \geq 2$ . Then, for any  $p_0$ , the last proposer's payoff in any robust outcome is at least  $1 - \frac{1}{M}(\frac{M+1}{M})$ .*

Theorem 7 tells us that, for robust outcomes in large groups, the last proposer receives nearly the entire prize. Notice that this result requires only two proposal rounds. From this, additional implications concerning the role of predictability in environments with pure redistributive politics are immediate. In settings where the recognition order and number of rounds are determined randomly, robust outcomes are near-dictatorial for large groups as long as individuals have minimal foreknowledge ( $K_1, K_2 \geq 1$ ).

## 6. APPLICATION TO DYNAMIC LEGISLATIVE POLICY MAKING

Throughout this paper, we have confined attention to static collective choice problems (the selection of the policy that will prevail at a given point in time).

<sup>18</sup>In addition to the results summarized in Theorem 6, it is also easy to show that when  $T \geq 2$  and  $i(T-1) = i(T)$ , the last proposer receives the entire prize (a payoff of unity).

In this section, we describe an extension to dynamic collective choice problems (the selection of policies for multiple periods, present and future).

Suppose the legislature must select a policy for each period  $z = 1, 2, \dots, Z$ , where  $Z$  is potentially large but finite. Suppose also that each legislator's preferences over policies are intertemporally separable (so that the outcome in one period does not affect preference orderings over policies in another period). The group convenes and deliberates before the first period and between each successive pair of periods. Each of these between-period meetings (a "session") consists of multiple proposal rounds. Every proposal specifies a policy for the upcoming period *and one for every subsequent period*. For example, if a period represents a year and the policy consists of a tax rate, a proposal specifies a rate for the upcoming year and one for every subsequent year.<sup>19</sup> Activity prior to round  $t$  of session  $z$  establishes default policies for period  $z$  onward. The initial defaults (in place for round 1 of session 1) are predetermined. If a proposal passes, it displaces the previous default; if it does not pass, the previous default is unchanged. At the end of session  $z$ , the final choice for period  $z$  is implemented. The final session  $z$  choice for any subsequent period carries over as the initial default for session  $z + 1$ .

Imagine that the number of proposal rounds in each session is sufficiently large and that the recognition orders are sufficiently inclusive. In that case, it is easy to solve the dynamic collective choice problem by backward recursion. By Corollary 1 (reported in Section 2.3), we know that the initial default for session  $Z$  is irrelevant. The period  $Z$  outcome is simply the policy selected by our static model of collective choice. In light of this observation, proposals in session  $Z - 1$  cannot affect session  $Z$  outcomes. Consequently, the period  $Z - 1$  outcome is also the policy selected by our static model of collective choice. Continuing the recursion, we see that the dynamic problem decomposes into a sequence of static problems, each of which can be treated separately.

Several conclusions follow. When feasible policies for each period belong to a pork barrel policy space that satisfies Assumptions A1–A3, the last proposer in each session acts as a dictator with respect to the policy for the associated period. In contrast, when feasible policies belong to a one-dimensional policy space and preferences are single-peaked, the preferences of the "median voter" prevail in every period.<sup>20</sup>

Note that these conclusions also hold when the recognition order for each session is determined randomly and revealed at the outset of each session. Moreover, neither the composition of the legislature nor the preferences of the legislators need remain constant from one session to the next. We do not require foreknowledge of any of these features "years" in advance; even without such knowledge, legislators know that the starting point for future sessions

<sup>19</sup>Failing to explicitly specify a tax rate for an upcoming year is equivalent to proposing the current default rate applicable to that year.

<sup>20</sup>This contrasts with a result in Baron (1996), which establishes only convergence over time to the median voter outcome in a dynamic setting.

is irrelevant, so they concern themselves only with the policy for the upcoming period. Our conclusions need to be modified, however, if the number of proposal rounds within each period is small (recall, however, Theorem 3) or if there are structural links between periods (e.g., the chosen policy alters a state variable that determines subsequent opportunities). A complete analysis of dynamic issues is beyond the scope of this paper.

It is useful to contrast our dynamic framework with the dynamic collective choice models of Baron (1996), Baron and Herron (1999), Epple and Riordan (1987), and Ingberman (1985). Although there are a number of differences, one deserves particular emphasis: Each of these papers assumes that, in each period, the legislature chooses the policy for that period and that this policy remains in place for future periods unless it is subsequently modified. In our framework, this amounts to imposing the restriction that a proposal must specify the *same* policy (e.g., the same tax rate) for all future periods. This means, for example, that legislators cannot propose phased-in changes and sunset clauses. Our approach does not impose such restrictions.

## 7. CONCLUSIONS

We have explored the nature of legislative policy making in institutions with two critical features: agenda setting occurs in real time and the default policy evolves. We have shown that a simple institution with these features always selects a Condorcet winner when one exists, provided a sufficient number of individuals can make proposals. Because the existence of a Condorcet winner is rarely guaranteed, we have focused the bulk of our attention on two widely studied policy spaces: one with pork barrel policies (including a natural variation) and the other that allows for flexible division of a fixed prize. In each instance, we found that the institution under consideration vests the final proposer with dictatorial or near-dictatorial power under relatively weak conditions. We have also shown that as long as the decision making body is large, the final proposer's preferences almost certainly prevail, even with a small amount of foreknowledge about the number of proposal rounds and the identity of future proposers. Thus, for the environments considered here, the combination of real-time agenda setting and evolving defaults tends to concentrate power in the hands of the legislator with the last word.

The institution examined here differs from real legislative institutions in many potentially important respects. It is natural to wonder how the specific rules and procedures observed in practice, but absent from the model, affect policy choices, and whether particular procedures effectively promote a more egalitarian distribution of political power for this class of institutions. Features of particular interest include amendment processes, endogenous closure rules, procedures for determining recognition orders, restrictions on allowable proposals, and supermajority requirements. Conceivably, various commonly observed rules and procedures may have come into use precisely because they counteract the forces that tend to concentrate political power.

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## APPENDIX

**PROOFS OF THEOREMS 3 AND 4:** We begin with some notation. Let  $l^n$  denote the project with the  $n$ th lowest cost, e.g.,  $l^M$  is the project with the median cost. Let  $H^n(p)$  denote the set of  $n$  highest cost projects within the policy  $p$  and let  $L^n(p)$  denote the set of  $n$  lowest cost projects within the policy  $p$ .

We now state some preliminary results with regard to the continuation mappings  $f_T$  and  $f_{T-1}$ . The proofs are either given in the text or left to the reader.

**RESULT 3.1:** For all  $p$ ,  $f_T(p)$  includes at most  $M$  projects and always includes project  $i(T)$ .

**RESULT 3.2:** Policy  $\{i(T)\}$  is a Condorcet winner in  $F_T$ .

**RESULT 3.3:** No project in the set  $H^{M-Q^*}(E \setminus \{i(T)\})$ , which consists of the  $M - Q^*$  highest cost projects other than  $i(T)$ , is ever included in  $f_T(p)$ .

**RESULT 3.4:** Whenever  $f_T(p)$  contains fewer than  $M$  projects, no project in the set  $H^{M-1}(E \setminus \{i(T)\})$  is included in  $f_T(p)$ .

**RESULT 3.5:** Whenever  $i(T)$  is the lowest cost project, no project in the set  $H^{M-1}(E)$  is ever included in the final policy.

**RESULT 3.6:** For any default  $p$ , the total cost of  $f_{T-1}(p)$  cannot exceed the total cost of  $f_T(p)$ .

**RESULT 3.7:** Whenever the default for either round  $T$  or round  $T - 1$  equals the policy  $L^{Q^*}(E)$ , which consists of the  $Q^*$  lowest cost projects, the final outcome equals  $\{i(T)\}$ .

We proceed by further characterizing the continuation mapping  $f_{T-1}$ . To do so, we define  $p^*$  as the favorite policy for player  $i(T - 1)$  within the continuation set  $F_T$ . Notice that  $p^*$  always includes project  $i(T)$ , and project  $i(T - 1)$  is excluded from every policy in  $F_T$  with a lower cost than  $p^*$ .

CLAIM 3.1: *For all recognition orders, we must have  $f_{T-1}(p) \subseteq p^*$  for all  $p$ . Let  $p$  be the default for round  $T - 1$ . Suppose first that the cost of the continuation policy  $f_T(p)$  is lower than the cost of  $p^*$ . From Result 3.6, this implies that project  $i(T - 1)$  can never be included in the final policy. Therefore, from Results 3.1 and 3.2, it follows that player  $i(T - 1)$ 's best response must lead to the final outcome  $\{i(T)\}$ , a subset of  $p^*$ . Now suppose that the cost of the continuation policy  $f_T(p)$  is higher than the cost of  $p^*$ . From Result 3.1 it follows that  $p^*$  is preferred over  $f_T(p)$  by player  $i(T)$  and by all the  $M - 1$  players whose projects are not included in  $f_T(p)$ . Therefore, player  $i(T - 1)$  can guarantee that  $p^*$  becomes the final outcome. Finally, if the cost of the continuation policy  $f_T(p)$  equals the cost of  $p^*$ , then  $f_T(p) = p^*$ , in which case  $i(T - 1)$  achieves  $p^*$  by proposing  $p$ .*

We now separate the analysis into three cases, according to the resulting identities of  $i(T - 1)$  and  $i(T)$ :

CLAIM 3.2: *Whenever  $i(T - 1) \in H^{M-Q^*}(E \setminus \{i(T)\})$ , we must have  $f_{T-1}(p) = \{i(T)\}$  for all  $p$ . Due to Claim 3.1, it suffices to show that  $p^* = \{i(T)\}$ , which follows from Results 3.1 and 3.3.*

CLAIM 3.3: *Whenever  $i(T - 1) \in L^{M-1}(E \setminus \{i(T)\})$ , we must have  $f_{T-1}(p) \subseteq \{i(T), i(T - 1)\}$  for all  $p$ . Due to Claim 3.1, it suffices to show that  $p^* = \{i(T), i(T - 1)\}$ . From Result 3.1, for this it suffices to show that the policy  $\{i(T), i(T - 1)\}$  belongs to the continuation set  $F_T$ . This follows from the fact that  $f_T(p) = \{i(T), i(T - 1)\}$  for  $p = \{i(T), i(T - 1)\} \cup H^{M-1}(E \setminus \{i(T)\})$ .*

CLAIM 3.4: *Whenever  $i(T - 1)$  is one of the  $Q^* - 1$  lowest cost projects in  $H^{M-1}(E \setminus \{i(T)\})$  and  $i(T)$  is the lowest cost project overall, we must have  $f_{T-1}(p) = \{i(T)\}$  for all  $p$ . Due to Claim 3.1, it suffices to show that  $p^* = \{i(T)\}$ , which follows from Results 3.1 and 3.5.*

For the first part of Theorem 3 (where  $T = 2$ ), notice that dictatorship arises whenever the conditions of Claims 3.2 or 3.4 are satisfied, but this occurs with probability  $\frac{M-Q^*}{N-1}$  (for Claim 3.2)  $+$   $\frac{1}{N} \frac{Q^*-1}{N-1}$  (for Claim 3.4)  $= \frac{1}{N-1}(M - Q^* + \frac{Q^*-1}{N}) = \frac{1}{2} - \frac{Q^*-1}{N}$ . For the second part of Theorem 3, we require two additional results:

CLAIM 3.5: *Whenever some player outside the set  $p^* \setminus \{i(T)\}$  is recognized prior to round  $T - 1$ , the final outcome must equal  $\{i(T)\}$ . From Claim 3.1 it follows that, among all policies in  $F_{T-1}$ , policy  $\{i(T)\}$  is the most preferred for any player outside the above set. The claim follows from Result 3.2 and Theorem 1.*

CLAIM 3.6: *Whenever  $p^* \subseteq \{i(T), i(T - 1)\}$  (as in Claims 3.2–3.4), we must have  $f_{T-2}(p) = \{i(T)\}$  for all  $p$ . Because  $i(T - 2) \neq i(T - 1)$ , the hypothesis of Claim 3.5 is necessarily satisfied for round  $T - 2$ , which delivers the result.*

For the second part of Theorem 3 (where  $T \geq 3$ ), notice that dictatorship will arise whenever the conditions of Claims 3.5 or 3.6 are satisfied. However, this occurs either if the conditions of Claims 3.2–3.4 are satisfied (so that Claim 3.6 holds) or if this is not the case but some player outside the set  $p^* \setminus \{i(T)\}$  is recognized prior to round  $T - 1$  (so that Claim 3.5 holds). The combined probability of these events is no less than  $1 - \frac{Q^*-1}{N}$  (for Claims 3.2–3.4)  $+ \frac{Q^*-1}{N} [1 - (\frac{M-2}{N-1})^{T-2}]$  (for Claims 3.2–3.4 not to hold while Claim 3.5 holds)<sup>21</sup>  $= 1 - \frac{Q^*-1}{N} (\frac{M-2}{N-1})^{T-2}$ , as stated in the theorem.

We now turn to Theorem 4. To characterize the continuation mapping  $f_{T-1}$ , we separate the analysis into four cases, according to the resulting identities of  $i(T - 1)$  and  $i(T)$ , both of which are known at the beginning of round  $T - 1$ .

CLAIM 3.7: *Whenever  $i(T - 1) \in H^{M-Q^*}(E)$ , we must have  $f_{T-1}(p) = \{i(T)\}$  for all  $p$ .* This follows from Claim 3.2.

CLAIM 3.8: *Whenever  $i(T - 1) = I^{M+Q^*-1}$  and  $i(T) \in H^{M-Q^*}(E)$ , we must have  $f_{T-1}(p) = \{i(T)\}$  for all  $p$ .* This also follows from Claim 3.2.

CLAIM 3.9: *Whenever  $i(T - 1) \in L^{M-1}(E)$ , we must have  $f_{T-1}(p) \subseteq \{i(T), i(T - 1)\}$  for all  $p$ .* This follows from Claim 3.3.

CLAIM 3.10: *Whenever  $i(T - 1) = I^M$ , we must have  $f_{T-1}(p) \subseteq \{i(T), i(T - 1), L^{M-2}(E)\}$ .* When player  $i(T)$  belongs to  $L^{M-1}(E)$ , this follows from Claim 3.3. Now suppose player  $i(T)$  belongs to  $H^{M-1}(E)$ . Due to Claim 3.1, it suffices to show that  $p^* = \{i(T), i(T - 1), L^{M-2}(E)\}$ . From Results 3.1 and 3.4, any policy in  $F_T$  that includes project  $i(T - 1)$  must have at least  $M$  elements and also include project  $i(T)$ . As a result,  $\{i(T), i(T - 1), L^{M-2}(E)\}$ , which is the cheapest such policy, is weakly preferred by player  $i(T - 1)$  over every policy in  $F_T$ . It remains to show that this policy actually belongs to  $F_T$ . This follows from the fact that  $f_T(p) = \{i(T), i(T - 1), L^{M-2}(E)\}$  for  $p = \{I^{M-1}\}$ .

Next, we derive some properties of the continuation mapping  $f_{T-2}$ . Notice that this mapping might be random from the viewpoint of round  $T - 2$  because the identity of  $i(T)$  may yet to be revealed. We separate the analysis into two cases, according to the resulting identities of  $i(T - 2)$  and  $i(T - 1)$ .

CLAIM 3.11: *Whenever  $i(T - 1) \in L^{M-1}(E)$ , we must have  $f_{T-2}(p) = \{i(T)\}$  for all  $p$ .* Consider the beginning of round  $T - 2$ , where the identity of  $i(T - 1)$  is already known. From Claim 3.9 it follows that the final policy must be a

<sup>21</sup>Nondictatorship can arise only if, for every period prior to  $T - 1$ , the proposer is not  $i(T)$ , and his project is included in  $p^*$ . Since  $p^*$  has at most  $M$  elements, at most  $M - 2$  eligible individuals satisfy this condition in each period (given that the same individual cannot propose twice in a row), and there are  $N - 1$  possible choices.

subset of  $\{i(T), i(T - 1)\}$ , regardless of the realization of  $i(T)$ . Indeed, from the viewpoint of round  $T - 2$ , the final outcome can be described as a lottery over this set. However, observe that among all possible lotteries over  $\{i(T), i(T - 1)\}$ , player  $i(T - 2)$ —together with every other player, except for  $i(T - 1)$ —strictly prefers the lottery that assigns probability 1 to project  $i(T)$ . Moreover, from Result 3.7, player  $i(T - 2)$  can guarantee that such a lottery is adopted by proposing policy  $\{L^{Q^*}(E)\}$ .

CLAIM 3.12: *Whenever  $i(T - 1) = l^M$  and  $i(T - 2) \in \{l^{M-1}\} \cup H^{M-1}(E)$ , we must have  $f_{T-2}(p) = \{i(T)\}$  for all  $p$ .* Consider the beginning of round  $T - 2$ . From Claim 3.10 it follows that the final policy must be a subset of  $\{i(T), i(T - 1)\} \cup L^{M-2}(E)$ . As before, from the viewpoint of round  $T - 2$ , the final outcome can be described as a lottery over such a set. Observe that among all possible lotteries over this set, player  $i(T - 2)$ —together with every other player in  $l^{M-1} \cup H^{M-1}(E)$ —strictly prefers the lottery that assigns probability 1 to project  $i(T)$ . Moreover, player  $i(T - 2)$  can guarantee that such a lottery is adopted by proposing policy  $\{L^{Q^*}(E)\}$ .

To conclude the proof, notice that whenever the hypotheses of Claims 3.7, 3.8, 3.11, or 3.12 are met, the final outcome will equal  $\{i(T)\}$ . These hypotheses are met with an ex ante probability equal to  $\frac{M-Q^*}{N}$  (for Claim 3.7) +  $\frac{1}{N} \cdot \frac{M-Q^*}{N-1}$  (for Claim 3.8) +  $\frac{M-1}{N}$  (for Claim 3.11) +  $\frac{1}{N} \cdot \frac{M}{N-1}$  (for Claim 3.12) =  $1 - \frac{1}{N} - \frac{Q^*-2}{N-1}$ . Q.E.D.

PROOF OF THEOREM 5: Let  $p^*$  denote the policy that consists of all  $i(T)$ 's projects and nothing else. As in the proof of Theorem 2, we proceed by establishing three claims.

CLAIM 5.1: *If the default policy for round  $T$  is  $p^*$ , then this is the outcome.* Agent  $i(T)$  can achieve this outcome simply by proposing the default policy; because this is his global optimum, it is his best achievable outcome.

CLAIM 5.2: *Regardless of the default policy for round  $T$ , the outcome never includes projects for more than  $M - 1$  individuals other than  $i(T)$ .* Suppose, on the contrary that, for some default, the outcome contains projects for  $M - 1 + m$  individuals other than  $i(T)$ , with  $m > 0$ . Identify any group of exactly  $M$  individuals, including  $i(T)$ , who weakly prefer the outcome to the default; call this group  $G$ . The outcome includes projects for at least  $m$  individuals who are not in  $G$ . Consider a new policy that is identical to the original outcome except that it deletes these projects. Members of  $G$  strictly prefer this new policy to the default (they strictly prefer it to the original outcome, which they weakly prefer to the default), so this proposal passes as well. Thus, the original proposal was not a best choice for  $i(T)$ .



CLAIM 5.3: *All policies in  $F_T$  other than  $p^*$  cost strictly more than  $p^*$ .* Choose any  $p \in F_T$  other than  $p^*$ . If  $p$  contains at least  $M - 1$  projects, the claim follows from Assumption A4. The remainder of this proof therefore focuses on the case where  $p$  contains fewer than  $M - 1$  projects.

Consider any  $q$  for which the round  $T$  continuation outcome is  $p$  (that is,  $f_T(q) = p$ ). We claim that  $q \neq p$ . Assume on the contrary that  $q = p$ . Whereas  $p \neq p^*$  by assumption, the following two possibilities are exhaustive.

CASE 1: Policy  $q$  contains only projects for  $i(T)$ , but does not contain all of  $i(T)$ 's projects. For this case, consider any  $p'$  that contains all of  $i(T)$ 's projects, one project for  $M - 1$  other individuals, and no projects for any other individual. By Assumption A5, all of the  $M$  individuals associated with the added projects, including  $i(T)$ , strictly prefer  $p'$  to  $q$ . However, then  $i(T)$  would gain by proposing  $p'$ , a contradiction.

CASE 2: Policy  $q$  contains at least one project for some  $j^* \neq i(T)$ . For this case, consider a policy  $p'$  constructed by eliminating from  $q$  all projects associated with  $j^*$ . All individuals but  $j^*$ , including  $i(T)$ , strictly prefer  $p'$  to  $q$ . However, then  $i(T)$  would gain by proposing  $p'$ , a contradiction.

Let  $G$  denote a group of  $M$  individuals including  $i(T)$ , all of whom strictly prefer  $p$  to  $q$  (such a group always exists by Assumption A1). Let  $G_0$  denote the set of individuals in  $G$  for whom  $p$  contains no project; because  $|p| < M - 1$ , we know that  $G_0$  is nonempty. Let  $\bar{G}$  denote all individuals not in  $G$ . By the same argument used in the proof of Claim 5.2, it follows that  $p$  does not include any projects for members of  $\bar{G}$ . Finally, for any  $x \in P$ , let  $c^x$  denote the total per capita cost of  $x$  (that is,  $\sum_{(i,r) \in x} c_{ir}$ ).

We now prove that  $c^p > c^{p^*}$  through a series of four steps.

STEP 1:  $c^q > c^p$ . Whereas  $q \neq p$ , we know that  $c^q \neq c^p$  by Assumption A1. If  $c^p > c^q$ , members of  $G_0$  would strictly prefer  $q$  to  $p$ , a contradiction.

STEP 2: We show that  $p$  includes at least one project for some  $j^* \in G$  other than  $i(T)$ . Suppose not. Because  $p$  does not include any projects for members of  $\bar{G}$  and because  $p \neq p^*$ , this means  $p$  contains only projects for  $i(T)$ , but does not contain all of  $i(T)$ 's projects. Consider  $p'$  that contains all of  $i(T)$ 's projects, one project for each other member of  $G$ , and no projects for any other individual. By Assumption A5, all members of  $G$ , including  $i(T)$ , strictly prefer  $p'$  to  $p$ ; because they strictly prefer  $p$  to  $q$  by construction, they also strictly prefer  $p'$  to  $q$ . However, then  $i(T)$  would gain by proposing  $p'$  instead of  $p$ , a contradiction.

STEP 3: We show that  $q$  includes at least one project for all  $j \in \bar{G}$ . Suppose on the contrary that  $q$  contains no project for some  $j' \in \bar{G}$ . Then, because  $c^q > c^p$ , this individual strictly prefers  $p$  to  $q$ . This means that  $p$  is strictly preferred to  $q$  by  $j'$  and all members of  $G$ , which constitutes a supermajority. Consider  $p'$  constructed by eliminating from  $p$  all projects associated with  $j^*$  (the individual identified in Step 2). Note that  $p'$  is strictly preferred to  $p$  by  $j'$  and all individuals in  $G$  other than  $j^*$ . Whereas  $p$  is strictly preferred to  $q$  by all members of this same group, a strict majority prefer  $p'$  to  $q$ . However, then  $i(T)$  would gain by proposing  $p'$ , a contradiction.

STEP 4: We show that  $p$  includes all of  $i(T)$ 's projects (from which it follows immediately that  $c^p > c^{p^*}$ , as desired). Suppose not. For any policy  $x \in P$  and individual  $j$ , define  $c_j^x$  as the total per capita cost of the projects in  $x$  for individual  $j$  (that is,  $\sum_{r=1}^{R_j} c_{jr} I_{jr}^x$ , where  $I_{jr}^x = 1$  if  $(j, r) \in x$  and 0 otherwise). Define the following two sets:

- Set  $A$  is all members of  $G$  (other than  $i(T)$ ) such that  $c_j^p < c_j^q - \bar{c}$ .
- Set  $B$  is all members of  $G$  other than  $i(T)$  and those in  $A$ .

Now construct a policy  $p'$  as follows: For  $i(T)$ , include all projects; for members of  $\bar{G}$ , include no projects; for each member of  $A$ , include all the projects in  $p$  and exactly one more; for each member of  $B$ , include all the projects in  $q$  and no others.

We claim next that  $p'$  is strictly less costly than  $q$ . This conclusion emerges from the following series of inequalities, each of which is justified below:

$$\begin{aligned}
 c^{p'} &= c_{i(T)}^{p'} + \sum_{j \in G, j \neq i(T)} c_j^{p'} \\
 &\leq \bar{R}\bar{c} + \sum_{j \in G, j \neq i(T)} c_j^q \\
 &\leq \bar{R}\bar{c} + \sum_{j \in G, j \neq i(T)} c_j^q + \left[ \sum_{j \in \bar{G}} c_j^q - (M-1)\underline{c} \right] + c_{i(T)}^q \\
 &= c^q - [(M-1)\underline{c} - \bar{R}\bar{c}] \\
 &< c^q.
 \end{aligned}$$

For the first inequality, we use three facts: First,  $i(T)$ 's projects cost no more than  $\bar{R}\bar{c}$ ; second, by construction,  $c_j^{p'} < c_j^q$  for all  $j \in A$ ; third, by construction,  $c_j^{p'} = c_j^q$  for all  $j \in B$ . The second inequality follows because  $c_{i(T)}^q \geq 0$  and, by Step 3,  $\sum_{j \in \bar{G}} c_j^q \geq (M-1)\underline{c}$ . The final inequality follows from Assumption A4.

Now we argue that  $i(T)$  and all members of  $A$  strictly prefer  $p'$  to  $p$ , and hence to  $q$  (because they also strictly prefer  $p$  to  $q$ ). For each member of this group, gross benefits are at least  $\underline{b}$  greater with  $p'$  than with  $p$  (because at least

one of their projects is added). Moreover,  $c^{p'} - c^p \leq (\bar{R} + M - 1)\bar{c}$  (this follows because  $p'$  adds at most  $\bar{R}$  projects for  $i(T)$  and one project for each member of  $A$ , and because, by construction,  $c_j^{p'} \geq c_j^p + \bar{c}$  for members of  $B$ ). Therefore, for members of this group, the net payoff is at least  $\underline{b} - (\bar{R} + M - 1)\bar{c}$  greater with  $p'$  than with  $p$ ; this difference is strictly positive by Assumption A5.

Because  $p'$  is strictly less costly than  $q$ , members of  $B$  also strictly prefer it to  $q$ . Thus, if  $i(T)$  proposes  $p'$ , it will pass, making  $i(T)$  better off—a contradiction. This concludes the proof of Claim 5.3.

Together, our three claims imply that  $p^*$  is a Condorcet winner in  $F_T$ . (The first claim establishes that  $p^*$  is in  $F_T$ . Consider any other  $p \in F_T$ . From our second claim,  $p$  contains no projects for at least  $M - 1$  individuals other than  $i(T)$ . By our third claim, all of these individuals strictly prefer  $p^*$  to  $p$ . Because  $p^*$  is  $i(T)$ 's most preferred policy, a majority prefer  $p^*$  to  $p$ .) Applying Theorem 1 completes the proof, exactly as in the proof of Theorem 2. *Q.E.D.*

PROOF OF THEOREM 6: A policy  $p$  is an  $N$ -dimensional vector, where the  $l$ th entry,  $p^l$ , corresponds to the share received by player  $l$ . Following this notation, let  $f_i^l(p)$  denote the share received by  $l$  under the continuation policy  $f_i(p)$ . We begin by characterizing  $f_T$ , for which we require some additional notation. Given a default policy  $p$ , identify some set  $G(p)$  that consists of  $M - 1$  players  $l$ , other than  $i(T)$ , with the  $M - 1$  lowest shares  $p^l$ . Let  $\lambda(p)$  denote the combined shares of these players (which cannot exceed  $\frac{1}{2}$ ).

CLAIM 6.1: For any default policy  $p$ ,  $f_T^{i(T)}(p) \geq 1 - \lambda(p)$ . Suppose to the contrary that there exists a  $p$  such that  $f_T^{i(T)}(p) < 1 - \lambda(p)$ . We show that there exists a proposal  $q$  that is strictly preferred by  $M$  players over  $p$  and that delivers a higher share than  $f_T^{i(T)}(p)$  for player  $i(T)$ , which is a contradiction. This proposal  $q$  is such that, for some positive  $\varepsilon$ , (i)  $q^l = p^l + \varepsilon$  for all  $M - 1$  players in  $G(p)$ , (ii)  $q^{i(T)} = 1 - \lambda(p) - (M - 1)\varepsilon$ , and (iii)  $q^l = 0$  for every other player. Notice that this proposal satisfies the above requirements for any  $\varepsilon$  such that  $1 - \lambda(p) - (M - 1)\varepsilon > f_T^{i(T)}(p)$ .

The first part of the theorem follows from Claim 6.1 and the fact that  $\lambda(p) \leq \frac{1}{2}$ .

CLAIM 6.2: For any default policy  $p$ , we must have  $f_T^l(p) \geq p^l$  for at least  $M - 1$  players other than  $i(T)$ . This follows from the fact that at least  $M$  players must weakly prefer  $f_T(p)$  over  $p$ .

Our next claim provides the full characterization of  $f_T$ :

CLAIM 6.3: For any default policy  $p$ , there exist two mutually exclusive sets of players,  $A(p)$  and  $B(p)$ , each consisting of  $M - 1$  players other than  $i(T)$ , such

that (i) for any players  $l \in A(p)$  and  $j \in B(p)$ , we have  $p^l \geq p^j$ , (ii)  $f_T^l(p) = 0$  for all  $l \in A(p)$ , and (iii)  $f_T^j(p) = p^j$  for all  $j \in B(p)$ . Furthermore,  $f_T^{i(T)}(p) \geq (M - 1)f_T^l(p)$  for any  $l \neq i(T)$ . This follows directly from combining Claims 6.1 and 6.2.

CLAIM 6.4: For any share  $\alpha \in [\frac{1}{2}, 1]$ , there exists a default policy  $p(\alpha)$  such that  $f_T^{i(T)}(p(\alpha)) = \alpha$ . Fix  $\alpha \in [\frac{1}{2}, 1]$ , let  $p(\alpha)$  satisfy  $p^{i(T)}(\alpha) = 2\alpha - 1$ , and let  $p^l(\alpha) = \frac{1}{N-1}(1 - p^{i(T)}(\alpha))$  for all remaining players. From Claim 6.3 it follows that  $f_T^{i(T)}(p(\alpha)) = p^{i(T)}(\alpha) + (M - 1)p^l(\alpha) = \alpha$ .

For the remainder of this proof, let  $\bar{p}$  denote the policy that delivers the entire prize to player  $i(T)$ .

CLAIM 6.5: For every  $p \neq \bar{p}$ , there is a continuation equilibrium for rounds  $T - 1$  and  $T$  such that if  $p$  is the round  $T - 1$  default, no player other than  $i(T - 1)$  and  $i(T)$  receives a strictly positive payoff. We prove this by construction. Define the policy  $q$  as follows:  $q^{i(T-1)} = \min\{\frac{1}{M}, 1 - f_T^{i(T)}(p)\}$ ,  $q^l = \min\{\frac{1}{M}, 1 - f_T^{i(T)}(p)\}$  for any other  $M - 1$  players (not including  $i(T)$ ), and  $q^{i(T)}$  is the residual. For round  $T$ , we select continuation equilibria as follows. When the round  $T$  default policy is  $q$ , player  $i(T)$  proposes leaving  $i(T - 1)$  with  $q^{i(T-1)}$  and keeping  $1 - q^{i(T-1)}$  for himself, and this passes. When the round  $T$  default policy is not  $q$ , any (equilibrium) continuation will do. Note that  $f_T^{i(T)}(q) = 1 - q^{i(T-1)} = \max\{1 - \frac{1}{M}, f_T^{i(T)}(p)\} \geq f_T^{i(T)}(p)$ . In round  $T - 1$ , resolve any voter's indifference in favor of  $i(T - 1)$ 's proposal if and only if the proposal would weakly benefit  $i(T)$ ; moreover, have  $i(T - 1)$  propose  $q$ . To demonstrate that this is an equilibrium, we must show that  $q$  passes, and that  $i(T - 1)$  cannot improve his payoff by making another proposal. The proposal  $q$  passes because at least  $M$  players ( $i(T)$  and members of  $A(p)$ ) vote in favor. We argue that  $i(T - 1)$  cannot improve his payoff in two steps.

STEP 1: We first show that  $f_{T-1}^{i(T-1)}(p) \geq f_T^{i(T)}(p)$ . Suppose player  $i(T - 1)$  proposes a policy  $p'$  such that  $f_T^{i(T)}(p') < f_T^{i(T)}(p)$ . From Claim 6.3, at least  $M - 1$  players receive a zero share under  $f_T(p')$  and, therefore, they vote against the proposal. Player  $i(T)$  will also vote against the proposal, implying that it cannot pass.

STEP 2: We have  $f_{T-1}^{i(T-1)}(p) \leq \min\{\frac{1}{M}, 1 - f_T^{i(T)}(p)\}$  (which immediately implies that  $i(T - 1)$  cannot make a better proposal than  $q$ ). Using Step 1, we have  $f_{T-1}^{i(T-1)}(p) \leq 1 - f_T^{i(T)}(p) \leq 1 - f_T^{i(T)}(p)$ . Moreover, from Claim 6.3,  $f_{T-1}^{i(T-1)}(p) \leq \frac{1}{M}$ .

CLAIM 6.6: Suppose the default for round  $T - 1$  is  $\bar{p}$ . Then, for any  $\alpha \in [0, \frac{1}{2}]$ , there is a continuation equilibrium in which  $i(T)$ 's payoff is  $\alpha$ . We prove this by

construction. Use any set of equilibrium continuations for round  $T$ . In round  $T - 1$ , resolve any voter's indifference in favor of  $i(T - 1)$ 's proposal if and only if the proposal is  $p(\alpha)$ ; moreover, have  $i(T - 1)$  propose  $p(\alpha)$ . To demonstrate that this is an equilibrium, first note that no proposal other than  $p(\alpha)$  passes in round  $T - 1$  (the default  $\bar{p}$  is  $i(T)$ 's favorite policy and at least  $M - 1$  other individuals, who receive zero if  $p(\alpha)$  passes, also vote for  $\bar{p}$ ). Second, note that all players but  $i(T)$  weakly prefer  $p(\alpha)$  to  $\bar{p}$ . This means that  $p(\alpha)$  passes and  $i(T)$  is willing to propose it.

CLAIM 6.7: *Suppose there exists a round  $t < T - 1$  such that  $i(t) \notin \{i(T), i(T - 1)\}$ . For all  $p$ , there exists a continuation equilibrium such that  $f_t^{i(T)}(p) = \alpha$ .* We prove this by construction. Use the continuation equilibria described in Claims 6.5 and 6.6, and consider the transformed (and equivalent) game with the reduced policy set  $F_{T-1}$  and only the first  $T - 2$  proposal rounds. The set  $F_{T-1}$  consists exclusively of policy  $f_{T-1}(\bar{p})$  (which grants  $i(T)$  a payoff of  $\alpha$ ) together with a collection of policies for which no player other than  $i(T)$  and  $i(T - 1)$  receives a strictly positive payoff. In every round of the transformed game, resolve any voter's indifference in favor of any alternative that leads to  $f_{T-1}(\bar{p})$ . Finally, for any default  $p$ , have  $i(t)$  propose  $f_{T-1}(\bar{p})$ . To demonstrate that this is an equilibrium, first note that every player but  $i(T)$  and  $i(T - 1)$  weakly prefer  $f_{T-1}(\bar{p})$  over every other element of  $F_{T-1}$ . This means that a majority weakly prefer  $f_{T-1}(\bar{p})$  over every other element of  $F_{T-1}$ .<sup>22</sup> By backward induction, it follows that, in every period, the default  $f_{T-1}(\bar{p})$  must lead to the outcome  $f_{T-1}(\bar{p})$  (in light of how we are resolving indifference).<sup>23</sup> Moreover, if the default in any period leads to something other than  $f_{T-1}(\bar{p})$  and if  $f_{T-1}(\bar{p})$  is proposed, it passes. Note that because the proposer in period  $t$  is neither  $i(T)$  nor  $i(T - 1)$ , he weakly prefers  $f_{T-1}(\bar{p})$  to all other possible continuation outcomes, so he is indeed always willing to propose it.

The second part of the theorem follows from Claim 6.7 and the hypothesis that at least three different players can make proposals. Q.E.D.

PROOF OF THEOREM 7: Fix  $\varepsilon \neq 0$  and consider the modified version of the game defined in the text. As in the proof of Theorem 6, let  $p^l$  denote the share

<sup>22</sup>When  $N \geq 5$ , this follows because the set of individuals other than  $i(T)$  and  $i(T - 1)$  constitutes a majority. When  $N = 3$ , we must choose the round  $T$  continuation equilibria so that  $i(T)$  always resolves indifference in favor of  $i(T - 1)$  when formulating his proposal. This means that  $i' \notin \{i(T), i(T - 1)\}$  receives a payoff of zero for all elements of  $F_{T-1}$ , but nevertheless continues to weakly prefer  $f_{T-1}(\bar{p})$  to all other elements of  $F_{T-1}$ . For any particular  $p \in F_{T-1}$ , either  $i(T - 1)$  or  $i(T)$  must weakly prefer  $f_{T-1}(\bar{p})$  to  $p$  (since their payoffs sum to unity in both cases), so a majority, including  $i'$ , weakly prefers  $f_{T-1}(\bar{p})$  to  $p$ , as required.

<sup>23</sup>In round  $T - 2$ , the default  $f_{T-1}(\bar{p})$ , if sustained, directly leads to the outcome  $f_{T-1}(\bar{p})$ . Consequently, it cannot be overturned by a proposal that does not lead to  $f_{T-1}(\bar{p})$ . Thus, if  $f_{T-1}(\bar{p})$  is the default in round  $T - 2$ , it is also the outcome. The same argument then applies to round  $T - 3$ , and so forth.

received by player  $l$  under policy  $p$ . We begin by deriving some properties of  $f_T$ . For any default  $p$ , player  $i(T)$ 's best response consists of expropriating the shares of  $M - 1$  players (other than  $i(T)$ ) with the highest values of  $p^l$ , while increasing the shares of the remaining  $M - 1$  players by  $\varepsilon$  (when  $\varepsilon > 0$ ), or of decreasing the shares of the remaining players by  $\min\{p^l, |\varepsilon|\}$  (when  $\varepsilon < 0$ ). The latter  $M - 1$  players will support  $i(T)$ 's proposal.

The following results are a consequence of this behavior. These results require some additional notation. Given  $p$ , let  $\mu(p)$  denote the maximum possible sum of shares  $p^l$  across any  $M - 1$  players other than  $i(T)$ . In addition, let  $\varepsilon_0 \equiv \max\{\varepsilon, 0\}$ .

RESULT 7.1: For every  $p$ ,  $f_T^{i(T)}(p) \geq \mu(p) - (M - 1)\varepsilon_0$ .

RESULT 7.2: For every  $p$ ,  $f_T^{i(T-1)}(p) \leq \frac{1}{M-1}\mu(p) + \varepsilon_0$ . This follows from the fact that either  $p^{i(T-1)} \leq \frac{1}{M-1}\mu(p)$  (and  $f_T^{i(T-1)}(p) \leq p^{i(T-1)} + \varepsilon_0$ ) or  $p^{i(T-1)} > \frac{1}{M-1}\mu(p)$  and, therefore,  $f_T^{i(T-1)}(p) = 0$ .

RESULT 7.3: For every  $p$ ,  $f_T^{i(T)}(p) \geq (M - 1)f_T^{i(T-1)}(p) - (N - 1)\varepsilon_0$ . This follows from combining Results 7.1 and 7.2.

RESULT 7.4: For every  $p$ ,  $f_T^l(p) = 0$  for at least  $M - 1$  players other than  $i(T)$ .

We separate the analysis of  $f_{T-1}$  into two cases, according to the sign of  $\varepsilon$ .

CASE I— $\varepsilon > 0$ :

CLAIM 7.1: For every  $p$ ,  $f_{T-1}^{i(T)}(p) \geq f_T^{i(T)}(p)$ . Given default  $p$ , suppose player  $i(T - 1)$  proposes a policy  $q$  such that  $f_T^{i(T)}(q) < f_T^{i(T)}(p)$ . Whereas  $\varepsilon > 0$ , every player such that  $f_T^l(q) \leq f_T^l(p)$  will vote against this proposal. This includes player  $i(T)$  together with every player such that  $f_T^l(q) = 0$ . However, from Result 7.4, this implies that at least  $M$  players will vote against the proposal and, therefore, it will fail.

CLAIM 7.2: Suppose  $p$  is such that  $f_T^{i(T)}(p) < \frac{M-1}{M}$ . Then  $f_{T-1}^{i(T-1)}(p)$  is bounded below by  $\frac{1}{M} - (M - 2)\varepsilon$ . Given default  $p$ , player  $i(T - 1)$  can achieve this payoff by proposing a policy  $q$  such that (i)  $q^{i(T-1)} = \frac{1}{M} - (M - 1)\varepsilon$ , (ii)  $q^{i(T)} = (M - 1)\varepsilon$ , (iii)  $q^l = 0$  for  $M - 2$  players such that  $f_T^l(p) = 0$  (which is possible due to Result 7.4), and (iv)  $q^l = \frac{1}{M}$  for the remaining  $M - 1$  players. (Notice that player  $i(T)$  together with all  $M - 2$  players in (iii) will support this proposal and  $f_T^{i(T-1)}(q) = q^{i(T-1)} + \varepsilon = \frac{1}{M} - (M - 2)\varepsilon$ .)

When  $p$  is such that  $f_T^{i(T)}(p) \geq \frac{M-1}{M}$ , it follows from Claim 7.1 that  $f_{T-1}^{i(T)}(p) \geq \frac{M-1}{M}$ . On the other hand, when  $p$  is such that  $f_T^{i(T)}(p) < \frac{M-1}{M}$ , from Result 7.3

and Claim 7.2 it follows that  $f_{T-1}^{i(T)}(p) \geq (M - 1)f_{T-1}^{i(T-1)}(p) - (N - 1)\varepsilon \geq \frac{M-1}{M} - M(M - 1)\varepsilon$ . In either case,  $f_{T-1}^{i(T)}(p) > 1 - \frac{1}{M}(\frac{M+1}{M})$  when  $\varepsilon$  is small.

CASE II— $\varepsilon < 0$ : Given any default  $p$ , let  $l^*(p)$  denote the player who receives the  $(M - 1)$ th lowest share other than  $i(T - 1)$  under the continuation policy  $f_T(p)$ . From Result 7.4, either  $f_T^{l^*(p)}(p) = 0$  or  $f_T^{i(T-1)}(p) = 0$  (or both). In either case,  $f_T^{l^*(p)}(p)$  cannot exceed  $\frac{1}{M-1}[1 - f_T^{i(T)}(p)]$ .

CLAIM 7.3: Suppose  $p$  is such that  $f_T^{i(T)}(p) \geq \frac{M-1}{M}$ . Then  $f_{T-1}^{i(T-1)}(p)$  is bounded below by  $\frac{1}{M-1}[1 - \frac{1}{M}(\frac{M+1}{M})] + 2\varepsilon$ . Given default  $p$ , player  $i(T - 1)$  can achieve this payoff by proposing a policy  $q$  such that (i)  $q^{i(T-1)} = \frac{1}{M-1}[1 - \frac{1}{M}(\frac{M+1}{M})] + \varepsilon$  (recall that  $\varepsilon < 0$ ), (ii)  $q^{l^*(p)} = \frac{1}{M(M-1)}$  (which is no smaller than  $f_T^{l^*(p)}(p)$ ), (iii)  $q^l = \frac{1}{M-1}[1 - \frac{1}{M}(\frac{M+1}{M})]$  (which is larger than  $\frac{1}{M(M-1)}$ ) for any  $M - 1$  players other than  $i(T)$ , (iv)  $q^{i(T)} = -\varepsilon$ , and (v)  $q^l = 0$  for the remaining  $M - 3$  players. (Notice that player  $l^*(p)$  together with every player such that  $f_T^l(p) = 0$  will support this proposal; from Result 7.4 and the definition of  $l^*$  there are at least  $M - 2$  such players other than  $i(T - 1)$  and  $l^*(p)$ . Moreover,  $f_T^{i(T-1)}(q) = q^{i(T-1)} + \varepsilon = \frac{1}{M-1}[1 - \frac{1}{M}(\frac{M+1}{M})] + 2\varepsilon$ .)

CLAIM 7.4: Suppose  $p$  is such that  $f_T^{i(T)}(p) < \frac{M-1}{M}$ . Then  $f_{T-1}^{i(T-1)}(p)$  is bounded below by  $\frac{1}{M} + 2\varepsilon$ . Given default  $p$ , player  $i(T - 1)$  can achieve this payoff by proposing a policy  $q$  such that (i)  $q^{i(T-1)} = \frac{1}{M} + \varepsilon$ , (ii)  $q^{i(T)} = -\varepsilon$ , (iii)  $q^l = 0$  for  $M - 2$  players such that  $f_T^l(p) = 0$  (which is possible due to Result 7.4), and (iv)  $q^l = \frac{1}{M}$  for the remaining  $M - 1$  players. (Notice that player  $i(T)$  together with all  $M - 2$  players in (iii) will support this proposal and  $f_T^{i(T-1)}(q) = q^{i(T-1)} + \varepsilon = \frac{1}{M} + 2\varepsilon$ .)

Observe that the lower bound for  $f_{T-1}^{i(T-1)}(p)$  from Claim 7.4 is larger than the lower bound from Claim 7.3, from which we can conclude that, for all  $p$ ,  $f_{T-1}^{i(T-1)}(p)$  is bounded below by  $\frac{1}{M-1}[1 - \frac{1}{M}(\frac{M+1}{M})] + 2\varepsilon$ . Moreover, by combining this lower bound with Result 7.3, we can conclude that, for all  $p$ ,  $f_{T-1}^{i(T)}(p)$  is bounded below by  $1 - \frac{1}{M}(\frac{M+1}{M}) + (N - 1)\varepsilon$ .

To complete the proof of the theorem, we combine Cases I and II: for  $\varepsilon \neq 0$  with  $|\varepsilon|$  sufficiently small and all  $p$ ,  $f_{T-1}^{i(T)}(p)$  is bounded below by  $1 - \frac{1}{M}(\frac{M+1}{M}) + (N - 1)\varepsilon$ , which converges to  $1 - \frac{1}{M}(\frac{M+1}{M})$  as  $|\varepsilon|$  converges to zero, thus establishing the desired result. Q.E.D.

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