In every society, present generations choose how much debt to pass to future generations and how much to invest in IPGs, such as public capital, pure research and development, and environmental preservation. This gives rise to a basic question in political economy: Are there institutions capable of protecting future generations from expropriation and of inducing optimal investment in IPGs? This institutional problem is challenging because future generations do not vote and the evidence suggests that present generations are imperfectly altruistic.\(^1\)

Although the prospects for future generations appear grim at first sight, previous work (described below) has shown that some institutions are capable of providing discipline through the capitalization of intergenerational spillovers. Selfish generations do not care directly about the impact of their actions on their descendants. However, they care about other variables that affect their own well-being, such as the price at which they will be able to sell their assets, or the future value of their social security benefits. An institution that is able to capitalize the intergenerational spillovers into one of these variables indirectly induces present generations to care about their descendants.

This paper studies how to protect future generations from expropriation and to induce optimal investment in IPGs by introducing constitutional restrictions on the tax base. The type of tax-base restrictions that we consider places limits on the tax instruments that the government can use to raise revenue, but not on the level of expenditures or debt. We show that the introduction of a constitutional amendment requiring that IPGs and debt be financed with land taxes makes intergenerational expropriation impossible and, for many cases of interest, induces optimal investment in IPGs. We also show that a weaker constitutional amendment requiring that IPGs be financed with land taxes, but imposing no restrictions on how to finance the debt, has a positive impact on IPGs, but not on expropriation. The paper also studies the political feasibility of these reforms. We show that the first reform is not politically feasible since it hurts current generations, but the weaker reform can induce a Pareto improvement. (JEL D1, D7, H0, H3, H4, H5, H6)

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1 See, for example, the studies of Joseph G. Altonji et al. (1992, 1997).
government can use to raise revenue, but not on the level of expenditures or debt. Tax-base restrictions are useful because they change the extent to which intergenerational spillovers are capitalized into the assets owned by current generations, and thus have a profound impact on their incentives.

We study these questions in the simplest possible environment containing all of the relevant mechanisms. We consider an endowment economy with two periods and two selfish generations. All the members of a generation are identical and decisions are made by majority rule. Generation 1 makes decisions in period 1 and generation 2 in period 2. As a result, any source of inefficiency in the model is due to a lack of intergenerational incentives. The first generation chooses how much to invest in an IPG and how much debt to issue. The IPG does not fully depreciate in the first period and also benefits the second generation. We assume that the second generation must repay the debt, and thus intergenerational redistribution is possible. Finally, the first generation owns a fixed amount of land that it sells to the second generation in a competitive market. The impact that the policy choices of the first generation have on the price of this land is the central mechanism at work in this paper.

We compare the performance of four institutions. First, there is a land-tax-only institution in which all of the revenue is collected using a tax per unit of land. Second, there is a head-tax-only institution in which all revenues are collected using an identical lump-sum tax on each individual. The head tax is meant to represent a combination of non-land taxes, from wage to dividend income taxes, which do not depend on land holdings. Third, there is a head-or-land-tax institution in which every generation chooses its own tax base, subject to the constraint that sufficient revenue needs to be raised in every period. Finally, there is a mixed institution in which expenditures in IPGs must be financed with land taxes, but in which there are no restrictions on the tax base used to pay for the debt. Since most national constitutions do not have the type of tax-base restrictions considered here, the head-or-land-tax institution represents the institutional status quo, and the other three institutions represent constitutional amendments that could be introduced.

We characterize the outcomes generated by the four institutions and show that changing the tax-base restrictions has a dramatic impact on intergenerational exchange. The land-tax-only institution makes redistribution through debt impossible and, in many cases of interest, induces optimal investment in IPGs. By contrast, the head-tax-only institution always generates as much expropriation as possible and inefficiently low levels of IPGs. The mixed institution generates an intermediate outcome: it is able to induce optimal investment in IPGs but not to stop redistribution through debt. Finally, the head-or-land-tax institution (i.e., the status quo case of no tax-base restrictions) generates multiple equilibria. There is a knife-edge equilibrium in which only land taxes are used, which replicates the outcome of the land-tax institution. But there are many other equilibria where head taxes are used in which present generations expropriate as much as possible and invest suboptimally in IPGs. Since land taxes are rarely used by central governments, the latter equilibria are the empirically relevant ones.

A comparison of these results shows that in our model the introduction of a constitutional amendment requiring IPGs and debt to be financed with land taxes (i.e., a move to a land-tax-only regime) makes intergenerational expropriation impossible and increases the incentives of present generations to invest in IPGs. A weaker constitutional amendment, requiring that IPGs be financed with land taxes but imposing no restrictions on how to finance the debt, has a positive impact on IPGs, but not on expropriation. Finally, a constitutional amendment restricting the use of land taxes, similar in spirit to California’s Proposition 13, reduces the incentives of current generations to invest in IPGs, but has no impact on the debt.

The paper also studies the political feasibility of these reforms. We show that a move to a land-tax base always decreases the welfare of present generations, and thus is unlikely to be politically feasible. We also show, however, that it is possible to move to a mixed institution and produce a Pareto improvement. Thus, this weaker reform is politically feasible.

The paper is related to several bodies of literature. First is the literature on the incidence of taxation, which studies who bears the burden of
exogenous tax changes. For example, Martin Feldstein (1977) shows that increases in future land taxes are immediately capitalized into land values. In this paper we take the results of this literature as our starting point and study how different tax base restrictions (as opposed to restrictions on the levels of taxes or expenditures) change the incentives to issue debt and to invest in IPGs. In other words, this paper studies how the known incidence properties of head and land taxes affect the politics of intergenerational exchange and constitutional reform.

Second, like the famous Henry George Theorem (see Anthony B. Atkinson and Joseph E. Stiglitz, 1980, Ch. 17), this paper shows that a restriction to a land-tax base has attractive properties. The relationship between the results stops there, however. Here the land-tax base is attractive because of its political economy properties. By contrast, the Henry George Theorem shows that in static economies, under some special conditions, the equilibrium land values equal the cost of providing the optimal level of the public good.

Third is the literature on the intergenerational properties of federalism. Wallace E. Oates and Robert M. Schwab (1988, 1996), Laurence Kotlikoff and Robert W. Rosenthal (1993), Edward L. Glaeser (1996), and Ronald I. McKinnon and Thomas J. Nechyba (1997) show that federalism can also protect future generations by inducing the capitalization of intergenerational spillovers into land values. There is, however, a crucial difference with this paper. This literature looks at decentralized institutions where interjurisdictional competition provides the capitalization force: future voters prefer jurisdictions with less debt and more local IPGs and thus bid up the price of land in those locations. By contrast, we study policy choices by national governments, where the mechanism generating the necessary capitalization cannot be competition across jurisdictions. Instead, this paper shows that appropriately chosen tax-base restrictions are sufficient to provide the necessary capitalization effects at the national level. Since central governments have historically issued large amounts of debt, and many public goods are national in nature (such as R&D or military capital), it is important to find ways to protect future generations without relying on federalism. The tax-base restrictions studied here are able to do just that.

Fourth, Rangel (2003) and Michele Boldrini and Ana Montes (1998) study investment in IPGs in economies without durable assets. They show that majority rule institutions generate equilibria in which “political capitalization” takes place: present selfish generations vote to invest in IPGs because otherwise future voters will cut their social security benefits. Unlike the land market capitalization studied here, “political capitalization” is sustained by cooperative equilibria in infinitely repeated games and, as a result, there are always bad equilibria in which the necessary incentives do not arise.

Finally, James M. Poterba (1994, 1995), James E. Alt and Robert Lowry (1994), Henning Bohn and Robert Inman (1995), and Roderick Kiewiet and Kristen Szakaly (1996) study budgetary institutions such as capital accounts. The goal of these institutions is also to protect future generations, but they work through a different mechanism. Instead of relying on “intergenerational capitalization,” they restrict the government’s ability to issue debt. For example, some institutions restrict the use of debt to the financing of concrete capital projects that have to be approved in a special referendum.

The paper is organized as follows. Section I presents the model of the economy and the institutions. Section II develops a preliminary result about the capitalization properties of the different tax bases that is useful in developing intuition. Section III characterizes the outcomes generated by the four institutions. Section IV studies the political economy of constitutional reform. Section V discusses limitations and extensions of our results. Section VI concludes.

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3 See also the Report of the President’s Commission on Capital Budgeting (1999).
I. Basic Model

A. Economy

Consider an economy with two periods and two homogeneous generations of size $N$. There are three goods: a private numeraire good, land, and an intergenerational public good (IPG). Land is a durable asset in fixed supply. To simplify the notation we assume that the total amount of land is equal to 1.

The timing of events is summarized in Figure 1. First, generation 1 is born at the beginning of period 1. Each member receives an endowment of $1/N$ units of land and $w$ units of the private good. Second, there is an election in which generation 1 chooses a level of expenditures in the IPG, denoted by $G_1$, and a level of debt, denoted by $D$. Third, generation 2 is born at the beginning of period 2. Each member is endowed with $w$ units of the private good but no land. At this time there is a land market in which generation 2 buys the land from generation 1. Immediately after, generation 1 consumes its wealth (including the proceeds from selling the land) and dies. Fourth, there is another election in which generation 2 chooses how many additional resources to spend in the IPG, denoted by $G_2$. Finally, generation 2 consumes all of its wealth and dies. Note that all private consumption takes place at the end of life.

We assume that generation 2 must repay the debt, i.e., it cannot escape intergenerational redistribution through default. Negative levels of debt represent public savings. We further assume that the government can borrow any amount up to an exogenous debt ceiling $D_{\text{max}}$. In order to insure that the second generation always has enough resources to repay the debt, we assume that $D_{\text{max}} < Nw$. The debt ceiling is needed to guarantee that the electoral problem of generation 1 is well defined. It can also be interpreted as a constitutional restriction on the size of the debt.

To simplify the analysis we do not model the financial sector explicitly. Instead we assume that the interest rate is constant and equal to one, and that the government borrows in the international market. This partial equilibrium assumption is fully justified for a small open economy. The production function for the public good is linear in both periods: it costs $\theta_i$ units of the private good to build one unit of the IPG in period $t$.

An IPG is a public good that has the following properties: it does not fully depreciate within one period; and future generations also care about the good. We model the first property by assuming that a fraction $\delta > 0$ of the IPGs purchased by the first generation remains usable in the second period. We model the second property by assuming that the amount of IPGs consumed by the second generation is $\delta G_1 + G_2$.

This formulation encompasses a wide class of public goods. As $\delta$ goes to zero, the IPG becomes two regular public goods, one in period 1 and one in period 2, with no intergenerational spillovers. The model also allows for technological progress, which is important for public goods such as infrastructure, space exploration, and environmental capital. This is captured by the case $\theta_1 > \theta_2$. Finally, the model...

---

4 Below we show that in several institutions generation 1 always expropriates as much as possible. In the absence of a debt ceiling it would try to set $D = \infty$.

5 As Auerbach et al. (1991) have shown, these types of debt ceilings are meaningful only in economies such as the one studied in this paper where the government does not have other fiscal instruments, such as pay-as-you-go social security, that permit intergenerational redistribution without explicitly issuing debt.
also includes the extreme case in which it is cost-effective to provide IPGs in the present, such as in the prevention of some types of environmental catastrophes. This is captured by the case \( \delta \theta_2 > \theta_1 \).

We assume that IPGs are nonreversible; that is, future generations can add to the IPG but not subtract from it. By contrast, an IPG is reversible if future generations can transform part of the IPG that they inherit into consumption. Investments in pure R&D are an example of a nonreversible IPG. Public buildings are an example of a reversible one. The extension to the case of reversible IPGs is discussed in Section VI.

There is no intergenerational altruism. The preferences of generation 1 are given by

\[
(1) \quad u(c) + v(l) + f(G_1)
\]

where \( c, l \) and \( G_1 \) denote, respectively, its consumption of the private good, land, and the IPG. The preferences of generation 2 are given by

\[
(2) \quad u(c) + v(l) + g(\delta G_1 + G_2)
\]

where \( c, l \) denote its consumption of the private good and land, and \( \delta G_1 + G_2 \) denotes its consumption of the IPG. \( u(\cdot), v(\cdot), f(\cdot) \) and \( g(\cdot) \) are twice continuously differentiable, strictly concave, and satisfy the Inada conditions.

Finally, let \( p \) denote the price of land, and

\[
(3) \quad \lambda(p|x) \equiv \arg \max_{l \geq 0} u(x - pl) + v(l)
\]

denote the demand for land of a member of generation 2 who has wealth \( x \). Also, let \( \bar{p} \) be the unique price satisfying the condition \( N\lambda(p|\bar{w}) = 1 \).\(^6\) This price will appear repeatedly in the analysis below.

### B. Pareto Optimality

In this section we characterize the set of Pareto optimal allocations in which all the members of a generation are given equal treatment. This characterization will be useful later on.

The set of Pareto optimal allocation with intragenerational equal treatment satisfies the following two properties:

(a) Private consumption and public expenditures are given by the solution to the following problem for some \( \alpha \in [0, 1] \):\(^7\)

\[
(4) \quad \max_{c_1, c_2, G_1, G_2 \geq 0} \alpha N(u(c_1) + f(G_1))
\]
\[
+ (1 - \alpha) N(u(c_2) + g(\delta G_1 + G_2))
\]

subject to

\[
(5) \quad Nc_1 + Nc_2 + \theta_1 G_1 + \theta_2 G_2 = 2Nw.
\]

(b) Every individual gets \( 1/N \) units of land.

This is a strictly convex problem and thus the FOCs, together with the feasibility constraint, provide a necessary and sufficient characterization of Pareto optimality. For \( \alpha \in (0, 1) \), the FOCs can be written as:\(^8\)

\[
(6) \quad \alpha u_1' = (1 - \alpha) u_2'
\]

\[
(7) \quad N \frac{g'}{u_2'} \leq \theta_2, \text{ with equality if } G_2 > 0
\]

and

\[
(8) \quad N \left( \frac{f'}{u_1'} + \delta \frac{g'}{u_2'} \right) = \theta_1.
\]

Conditions (6) and (8) hold with equality because, by the Inada conditions, for \( \alpha \in (0, 1) \) any Pareto optimal allocation is interior. Condition (7) need not hold with equality because, since generation 2 inherits \( \delta G_1 \) units of the IPG, its allocation can be interior even if \( G_2 = 0 \).

\(^6\) The existence and uniqueness of such a price are proven in the Appendix.

\(^7\) Land is not part of the following maximization problem because, by equal treatment, every individual gets to consume \( 1/N \) units of land.

\(^8\) To simplify the notation, subscripts in this paper always indicate periods or generations.
The first equation is a redistributional condition that pins down the allocation of private goods for any welfare weight. The second inequality is a Samuelson condition for the optimality of public investment in period 2; the left-hand side is the marginal benefit for generation 2 (measured in units of the private good) of an additional unit of expenditures, and the right-hand side is the marginal cost. Finally, equation (8) is the Samuelson condition for IPG expenditures in period 1. It takes into account that an additional unit of $G_1$ benefits both generations. The benefit for generation 2 is discounted because only a fraction $\delta$ of the period-1 investment survives.

By (7) and (8), when $G_2 > 0$ the Samuelson condition for $G_1$ can be rewritten as

$$N \frac{f'}{u_1} + \delta \theta_2 = \theta_1.$$  

This modified condition has a nice economic interpretation that will be useful in developing intuition later on. When $G_2 > 0$, bequeathing $\delta$ additional units of IPGs to generation 2 has the same impact on their behavior and well-being as bequeathing them the amount of private goods necessary to produce them. In either case, the “full income” of generation 2 increases by $\delta \theta_2$ units of the private good. As a result, as long as $G_2 > 0$, we can think of the intergenerational spillover as affecting the “full income” of generation 2, instead of its consumption of IPGs. By contrast, when $G_2 = 0$ the equivalence breaks down. As can be seen from (7), in that case the marginal cost of the IPG exceeds its marginal benefit, and generation 2 would not use the additional income to purchase the IPG.

C. Institutions

Political decisions are made by majority rule voting over the set of feasible policies, or by any other institution, such as Downsian competition, that selects Condorcet winners as the unique equilibrium outcomes whenever they exist. Since agents are homogenous, the existence of a Condorcet winner is trivially guaranteed: it is equal to the preferred policy of the representative voter. The politics become slightly more complex in Section IV, where we discuss extensions to the case of heterogenous agents.

We consider four constitutional regimes that differ only with respect to the tax base restriction that they impose on the government. First, there is a land-tax-only institution, in which all revenue is collected using a tax per unit of land. Since the size of the land stock is 1, the land taxes in periods 1 and 2, as a function of the policy choices, are given by

$$\tau_1^L(G_1, D) = \theta_1 G_1 - D$$

and

$$\tau_2^L(G_2, D) = \theta_2 G_2 + D.$$  

Note that the land tax can be negative. For example, if in period 1 the size of the debt is larger than the cost of producing the public good, the additional revenue is returned to the first generation using the land tax. In other words, the tax-base restrictions apply to any positive or negative transfers between the government and the citizenry.

Second, there is a head-tax-only institution in which all revenues are collected using an identical lump-sum tax from each individual. The head taxes in periods 1 and 2, as a function of the policy choices, are given by

$$T_1^H(G_1, D) = \frac{\theta_1 G_1 - D}{N}$$

and

$$T_2^H(G_2, D) = \frac{\theta_2 G_2 + D}{N}.$$  

Given that agents are homogenous, the head tax is meant to represent a combination of non-land taxes, from wage to dividend income taxes, in which the tax bill does not depend on land holdings. The head-tax-only institution provides a useful benchmark case for understanding the mechanisms at work in the paper. It is also motivated by some existing tax-base restrictions, such as Proposition 13 in California, which place limitations on the amount of revenue that can be raised using land taxes.

Third, there is a head-or-land-tax institution in which every generation chooses its own tax base,
subject to the constraint that sufficient revenue needs to be raised in every period, that is,

\[ \tau^M_1(G_1, D) + NT^M_1(G_1, D) = \theta_1 G_1 - D \]

and

\[ \tau^M_2(G_2, D) + NT^M_2(G_2, D) = \theta_2 G_2 + D. \]

Note that in this institution each generation chooses its own tax base, i.e., generation 1 cannot impose a tax base on generation 2. Since the type of tax-base restrictions studied in this paper are rare in the real world, the head-land-taxes institution is the one that best approximates existing constitutional regimes, and thus it can be interpreted as the institutional status quo.

Finally, there is a mixed institution which requires that expenditures in IPGs be financed with land taxes, but which places no restrictions on how to finance the debt (i.e., the debt can be financed with any combination of land and head taxes). In this case the constraints are given by

\[ \tau^L_k(G_k, D) \geq \theta_k G_k \]

and

\[ \tau^L_k(G_k, D) + NT^L_k(G_k, D) = \theta_k G_k + D. \]

and

\[ \tau^H_k(G_k, D) \geq \theta_k G_k \]

and

\[ \tau^H_k(G_k, D) + NT^H_k(G_k, D) = \theta_k G_k - D. \]

D. Equilibrium

Before providing a formal definition of equilibrium we need to define a few additional objects. The superscript \( k = L, H, HL, \) or \( M \) denotes the institution under consideration. \( (G_1, D) \) is a state variable summarizing the actions of the first generation. A full description of the outcomes generated by this institution requires specifying, for every state, a capitalization function \( p^k(G_1, D) \) describing the equilibrium price of land, and a policy response function \( G^k_2(G_1, D) \) describing the expenditures in IPGs in period 2. \((G^k_1, D^k)\) denotes the policy chosen by the first generation. Finally, \( G^k_2(G^k_1, D^k) \) denotes the policy chosen in equilibrium by generation 2.

An equilibrium in any of the institutions is given by a list \((p^k(\cdot), G^k_2(\cdot), \tau^k_2(\cdot), T^k_2(\cdot))\) specifying land prices, policy choices, and taxes in every state in period 2, and a list \((G^k_1, D, \tau^k_1, T^k_1)\) specifying policy choices and taxes in period 1, which satisfy the following properties:

(a) Political equilibrium in period 2: For every state \((G_1, D)\), the policy and tax choices \( G^k_2(G_1, D), \tau^k_2(G_1, D), \) and \( T^k_2(G_1, D) \) maximize the welfare of the representative member of generation 2, and \( \tau^k_2(G_1, D) \) and \( T^k_2(G_1, D) \) satisfy the tax-base restrictions of institution \( k \). At this stage the land market has already closed and voters take land holdings and net resources as fixed.

(b) Land market equilibrium: Generation 2 has rational expectations and anticipates perfectly the policy and taxes \((G^k_2(G_1, D), \tau^k_2(G_1, D), T^k_2(G_1, D))\) that will be chosen later in the period. In every state \((G_1, D)\) the equilibrium price \( p^k(G_1, D) \) satisfies:

\[ N\lambda(p^k(G_1, D) + \tau^k_2(G_1, D)) \]

\[ w - T^k_2(G_1, D) = 1. \]

(c) Political equilibrium in period 1: Generation 1 has rational expectations and anticipates perfectly the impact that its choices will have on the equilibrium price of land. The policy and tax choices \( G_1, D, \tau_1^k, \) and \( T_1^k \) maximize the welfare of the representative member of generation 1, taking as given the capitalization function \( p^k(\cdot) \) and satisfy the tax-base restrictions of institution \( k \).

Note several important features of this definition. First, the land market is competitive, and buyers and sellers take land prices as given. Since the supply of land is inelastic, the land market clearing condition in every state is given by (16).

Second, when members of generation 2 choose how much land to buy, they take as given the future outcome of the election. This is
justified since, with sufficiently large electorates, individuals do not believe that they can be pivotal. Their demand for land takes into account that, given the anticipated taxes, their after-tax wealth is $w - T^L_2(G_1, D)$, and the total cost of a unit of land is $\rho^L(G_1, D) + T^L_2(G_1, D)$.

Third, at the time of the election in period 1, generation 1 has rational expectations about the impact of their policy choice on land prices. In this model, this is the only mechanism through which current generations internalize the impact of their actions on future generations.

Fourth, in the land-tax-only and head-tax-only institutions, the choice of the tax base is trivial. In the former all taxes are financed with land taxes (i.e., $T^L_1(\cdot) = 0$); in the latter all taxes are financed with head taxes (i.e., $T^H_1(\cdot) = 0$). By contrast, in the head-or-land-tax and mixed institutions the electorate chooses every period how much revenue to raise with each base.

E. Discussion

In order to build a tractable framework we have made several strong assumptions that are worth discussing at the outset.

First, we have assumed that generation 2 must repay the debt, and that the financial markets allow the government to borrow at a constant interest rate any amount up to the debt ceiling. This rules out three important mechanisms limiting intergenerational redistribution. First, future generations can always repeal excessive amounts of debt. Second, anticipating this, the financial sector imposes limits on the amount that it is willing to lend.\(^9\) Third, the cost of borrowing increases with the size of the debt. The justification for our assumption is that while these mechanisms limit the size of the debt, they do not seem to stop all intergenerational redistribution. In this light, the debt ceiling can be thought of as a simple way of modelling the impact of these additional mechanisms, and the paper as a study of the extent to which tax-base restrictions can stop the redistribution that is still possible.

Second, we have assumed that generation 2 cannot expropriate the land and resources of generation 1. Although the possibility of intergenerational “warfare” clearly affects the incentives of present generations, casual observation suggests that it is not sufficient to eliminate intergenerational expropriation or to induce optimal investment in IPGs. Thus, as before, the paper studies the additional incentives that can be provided by tax-base restrictions.

Third, we have assumed that all of the members of a generation are identical. We do this to isolate the intergenerational incentive issues. It is well known that with heterogeneous preferences, voting rarely leads to an optimal choice of public goods. By eliminating this source of “political failure” we make sure than any inefficiencies are due to insufficient intragenerational incentives.

Fourth, we have assumed that there are only two periods and two generations. This is done to make the analysis tractable. Although the insights developed below suggest that the results should generalize to a full OLG economy, such an extension is beyond the scope of this paper.

Fifth, we have assumed that preferences are additively separable. This is useful for two reasons. First, it guarantees that all goods are normal. Second, it makes generation 2’s demand for land independent of its level of IPG consumption. Both properties are useful in deriving the results. The extension to the non-additively-separable case is discussed in Section VI.

Sixth, we have assumed that present generations are selfish. The assumption is made to make sure that the institutions that we study can provide the necessary incentives in the most challenging case.

II. Preliminary Result: The Impact of Tax-Base Restrictions on Capitalization

In the next section we show that constitutional tax-base restrictions have a significant effect on intergenerational exchange: they change the size of the debt, the amount invested in IPGs by the first and second generations, and the total amount of IPGs consumed by generation 2. As we will see, these differences result from the fact that the tax base affects the extent to which the choices of the first generation are capitalized into land prices.

\(^9\) See Dwight M. Jaffee and Thomas Russell (1976) for one of the first models on credit rationing by lenders.
In order to develop intuition and understand better the forces at work, it is useful to begin by studying the capitalization properties of the four institutions in isolation. We do this by characterizing the equilibrium price of land for any exogenously given policy function by generation 2, $G_2(G_1, D)$. Also, whenever generation 2 has a choice between head and land taxes, we assume that the fraction of those expenditures that is financed with land taxes is constant across states. More concretely, let $\phi_2$ denote the fraction of total expenditures that is financed with land taxes in the head-or-land-taxes institution, and let $\mu_2$ denote the fraction of debt expenditures that is financed with land taxes in the mixed institution.

Let $E(G_1, D) = D + \theta_2 G_2(G_1, D)$ denote the implied level of public expenditures of generation 2. The following result characterizes the equilibrium prices in the four institutions for any $G_2(\cdot)$, $\phi_2$, and $\mu_2$:

**PROPOSITION 1:** Suppose that generation 2 chooses $G_2(G_1, D)$, and that whenever generation 2 has a choice between head and land taxes, a constant fraction of those expenditures is financed with land taxes. The equilibrium land prices satisfy:

(i) For $k = L, H, HL$, $p^k(G_1, D)$ depends on the policy choices of the first generation only to the extent that they affect $E(G_1, D)$, i.e., only public expenditures in period 2 are capitalized;

(ii) In the land-tax-only institution public expenditures in period 2 are fully capitalized: $p^L(G_1, D) = \bar{p} - E(G_1, D)$;

(iii) In the head-tax-only institution public expenditures in period 2 are partially capitalized: $\frac{\partial p^H}{\partial E} \in (-1, 0)$;

(iv) In the head-or-land-tax institution, public expenditures in period 2 are partially capitalized unless only land taxes are used: $\frac{\partial p^{HL}}{\partial E} \in (-1, -\phi_2)$;

(v) In the mixed institution the composition of public expenditures matters, expenditures on $G_2$ are fully capitalized, and debt expenditures are partially capitalized: $\frac{\partial p^M}{\partial G_2} = -\theta_2$ and $\frac{\partial p^M}{\partial D} \in (-1, -\mu_2)$.

This result provides two useful insights. First, in all of the institutions the actions of generation 1 are capitalized only if they affect future expenditures. In particular, in order to be capitalized, an additional unit of $G_1$ must change the amount spent on IPGs in period 2 (i.e., it must change $G_2$). Since an additional unit of IPGs always increases the well-being of the second generation, this implies that some intergenerational spillovers might not be capitalized. Second, full capitalization of future public expenditures requires that only land taxes be used, regardless of whether or not this is required by the tax-base restrictions.

The intuition for this result follows directly from the conditions for land market equilibrium for the different institutions. Consider first the land-tax-only case. The land market clearing condition is given by

\[(17) \quad N\lambda(p^L(G_1, D) + \tau^L(G_1, D)|w) = 1\]

where the left-hand side denotes the total demand for land, the right-hand side is the inelastic supply, and $\tau^L(G_1, D) = E(G_1, D)$ the tax per-unit of land that is needed to finance the public expenditures in period 2. Note that, by the additive separability of preferences, the level of IPGs consumed by generation 2 does not affect the value that they place on land. As a result, $G_1$ and $D$ affect the demand for land only to the extent that they change the public expenditures of the second generation, and thus the size of the land tax. Furthermore, since all expenditures are financed with land taxes, the choices of the first generation affect the total cost of owning land, but not the wealth of the second generation. By the definition of $\bar{p}$, it follows that the total cost of land is given by

\[(18) \quad p^L(G_1, D) + \tau^L(G_1, D) = \bar{p}\]

which implies that

\[(19) \quad p^L(G_1, D) = \bar{p} - E(G_1, D).\]

Note that each additional unit of public expenditures lowers land values by exactly one unit.
Thus, public expenditures are fully capitalized. This part of the result is a straightforward replication of a well-known tax incidence result in the intergenerational context: when an asset is inelastically supplied, any future taxes on that asset are capitalized into current asset prices.

Now consider the case in which only head taxes are allowed. The land market clearing condition for this case is given by

\[ N\lambda(p^H(G_1, D)|w - T^H(G_1, D)) = 1 \]

where \( T^H(G_1, D) = E(G_1, D)/N \) denotes the head tax necessary to finance the public expenditures in period 2. In contrast to the previous case, each additional unit of expenditures reduces the wealth of the second generation by \( 1/N \), but has no impact on the cost of owning land. Again, note that \( G_1 \) and \( D \) affect the land demand of generation 2 only to the extent that, by changing public expenditures in period 2, they affect after-tax wealth. Since all of the goods are normal, giving an additional unit of wealth to generation 2 increases its land expenditures by less than unit. As a result, the expenditures are partially capitalized and \( \partial p^H/\partial E \in (-1, 0] \). In the extreme case of quasi-linear preferences, where the demand for land is not affected by wealth, we get zero capitalization.\(^{10}\)

The difference between head and income taxes resides in how they affect the demand for land. Head taxes operate through an income effect and are imperfectly capitalized. Land taxes operate through a price effect and are fully capitalized.

Next consider the case in which either tax base is allowed. Clearly, if \( \phi_2 = 1 \), the institution is equivalent to the land-tax-only case and full capitalization occurs. Suppose that \( \phi_2 < 1 \); the land market clearing condition is then given by

\[ N\lambda(p^{HL}(G_1, D) + \tau^{HL}(G_1, D)|w - T^{HL}(G_1, D)) = 1 \]

with \( \tau^{HL}(G_1, D) = \phi_2 E(G_1, D) \) and \( T^{HL}(G_1, D) = (1 - \phi_2) E(G_1, D)/N \). This is a combination of the previous two cases. For the same reasons as before, the fraction of expenditures \( \phi_2 \) that is financed with land taxes is fully capitalized; thus, \( \partial p^{HL}/\partial E \leq -\phi_2 \). The fraction of expenditures \( 1 - \phi_2 \) that is financed with head taxes is only partially capitalized, and thus \( \partial p^{HL}/\partial E > -1 \).\(^{11}\) We can conclude that the capitalization properties of the head-or-land-tax institution depend on the relative weight given to the two tax bases in the second period. The institution can replicate the capitalization properties of the land-tax-only case, but only if generation 2 never chooses head taxes.

Finally, consider the mixed institution, which is a bit different. If the electorate chooses to finance a fraction \( \mu_2 \) of the debt expenditures with land taxes, the market clearing condition is given by

\[ N\lambda(p^M(G_1, D) + \tau^M(G_1, D)|w - T^M(G_1, D)) = 1 \]

where \( \tau^M(G_1, D) = \mu_2 D + \theta_2 G^M_2(G_1, D) \) and \( T^M(G_1, D) = (1 - \mu_2)(D/N) \). For the same reasons as before, the land taxes are fully capitalized, while the head taxes are not. It follows that expenditures in period-2 IPGs are fully capitalized, since they increase land taxes one for one, but expenditures in the debt are fully capitalized only if \( \mu_2 = 1 \). In that case, this institution is equivalent to the land-tax regime. To conclude, note that in the mixed institution the composition of public expenditures in period 2 matters because it changes the relative size of the land and head taxes.

III. The Impact of Tax-Base Restrictions on Policy Choices

Proposition 1 provides a full characterization of the equilibrium land prices for any exogenously given policy function for generation 2. In this section we endogenize the policy choices

\(^{10}\) With quasi-linear preferences \( a(c) = c \), the equilibrium price of land is given by \( p = v(1/N) \), which is independent of the individual’s wealth.

\(^{11}\) In the case of quasi-linear preferences \( \partial p^{HL}/\partial E = -\phi_2 \).
and provide a complete characterization of the outcomes generated by the four institutions.

It is useful to begin the analysis with a general discussion of the common forces at work. Consider first the election in period 2. For any institution \( k \), and state \((G_1, D)\), the problem of the representative agent of generation 2 can be written as

\[
\max_{G_2 \geq 0} u\left(w - p^k(G_1, D) \frac{1}{N} - \theta_2 G_2 + D\right) + g(\delta G_1 + G_2). 
\]

Note that at the time of the election land prices are fixed since the land market has already taken place, and that each identical individual owns \(1/N\) units of land. It follows that regardless of the tax base, each individual pays \(1/N\)-th of the taxes. The outcome of the election, \(G_2^k(G_1, D)\), satisfies the FOC

\[
N \frac{g'}{u_2} \leq \theta_2, \text{ with equality if } G_2^k(G_1, D) > 0. 
\]

This is identical to the Samuelson condition for \(G_2\) derived in (7). It follows that the choice of \(G_2^k\) is Pareto optimal in every state. This is not surprising. The choice of \(G_2\) generates no intergenerational spillovers, and when voters have identical preferences, majority-rule voting leads to optimal public choice.

It is worth emphasizing that although the choice of the tax base at time 2 plays a crucial role in the capitalization of intergenerational spillovers, at the time of the election not much is at stake for generation 2. This is true only because the voters of each generation have identical preferences, wealth, and land holdings. As we will see below, the introduction of heterogeneity changes this.

Now consider the election in period 1. For any institution \( k \), the problem of the representative agent of generation 1 can be written as

\[
\max_{G_1 \geq 0, D \leq D_{\text{max}}} u\left(w - \frac{\theta_1 G_1 - D}{N}\right) + p^k(G_1, D) \frac{1}{N} + f(G_1). 
\]

The amount of revenue that the government needs to raise is now given by \(\theta_1 G_1 - D\). If \(D > \theta_1 G_1\), the surplus is returned to the individuals. As before, since every voter owns \(1/N\)-th of the land, each individual pays \(1/N\)-th of the taxes regardless of the tax base.

Expression (25) clearly illustrates that the properties of the capitalization function \(p^k(G_1, D)\) determine the extent to which generation 1 internalizes the impact of its actions on the second generation. Consider the choice of how much debt to issue. For the generation as a whole, the marginal benefit of issuing debt (measured in units of the private good) is 1. The marginal cost equals the loss in land value \(\partial p^k/\partial D\). It follows that in an institution in which \(\partial p^k/\partial D > -1\), generation 1 expropriates as much as possible by setting \(D^k = D^\text{max}\). The intuition is simple. Each additional unit of debt increases the after-tax wealth of the generation by one unit and decreases the value of its land by less than one unit. By contrast, in an institution in which \(\partial p^k/\partial D = -1\), any level of debt is an equilibrium. Generation 1 knows that the gain from each additional unit of debt is fully offset by the decrease in land values. It follows that the ability of the institutions to preclude intergenerational expropriation depends on the value of \(\partial p^k/\partial D\).

Now consider the choice of how much to invest in IPGs. The marginal benefit for generation 1 as a whole of investing in \(G_1\) (measured in units of the private good) is given by

\[
N \frac{f'}{u_1} + \frac{\partial p^k}{\partial G_1}. 
\]

The marginal cost is given by \(\theta_1\). By the Inada conditions, generation 1 always chooses a positive amount \(G_1^*\) of IPGs. Thus, if \(p^k(\cdot)\) is differentiable at the optimal choice, the following FOC must be satisfied:12

12 The technical qualifier is needed because, as we will see below, the capitalization function is differentiable almost everywhere, but not everywhere. This introduces some technical complications without changing the results. The intuition discussed here assumes that \(p^k(\cdot)\) is differentiable in equilibrium (i.e., at \((G_1^*, D^*)\)). This is generically true when \(\theta_1 < \delta \theta_2\). A full treatment of all the cases is provided in the proofs in the Appendix.
(27) \[ \theta_1 = N \frac{f'}{u_1} + \frac{\partial p^k}{\partial G_1}. \]

Compare this with the Samuelson condition for \( G_1 \), which, for the convenience of the reader, we reproduce here:

(28) \[ \theta_1 = N \frac{f'}{u_1} + \begin{cases} \theta_2 \delta & \text{if } G_2^k > 0 \\ N \frac{g'}{u_2} \delta & \text{if } G_2^k = 0. \end{cases} \]

Given that the Samuelson condition for \( G_2 \) is satisfied, this condition is necessary and sufficient for the Pareto optimality of the equilibrium allocation. A comparison between (27) and (28) shows that generation 1 chooses an efficient level of IPGs only if the magnitude of the capitalization effect is just right. In particular, efficiency requires that

(29) \[ \frac{\partial p^k(G_1^k, D^k)}{\partial G_1} = \begin{cases} \theta_2 \delta & \text{if } G_2^k > 0 \\ N \frac{g'}{u_2} \delta & \text{if } G_2^k = 0. \end{cases} \]

In other words, the capitalization function must be such that, in equilibrium, the marginal capitalization of \( G_1 \) is exactly equal to the marginal intergenerational spillover that it generates. Any less capitalization generates Pareto inefficiently low levels of \( G_1 \). Any extra capitalization generates Pareto inefficient excess expenditures. Note that, perhaps surprisingly, Pareto optimality requires full marginal capitalization in equilibrium, but not at every \((G_1, D)\). This is fortunate since, as we will see, none of the institutions is able to satisfy the stronger property.

An analogy with the well-understood problem of Pigouvian taxation might be useful. It is well known that, in the presence of positive externalities, the introduction of a Pigouvian tax equal to the marginal value of the externality at the optimum induces optimal behavior. A well-known problem with this policy instrument is that it requires a lot of information on the part of the government, which must be able to compute the optimal allocation and the size of the marginal spillovers. Part of the beauty of the institutions studied in this paper is that, through the capitalization of IPGs, they endogenously generate something analogous to a Pigouvian subsidy for the median voter of the first generation. Not only that, generation 1 receives these incentives through the “invisible hand of the market.” No information on the part of the government is required.

Although Proposition 1 tells us a lot about the capitalization properties of the three institutions, it does not provide a full characterization of \( \partial p^k/\partial G_1 \) and \( \partial p^k/\partial D \), which, as we have seen, are the features that generation 1 cares about. To see why, note that by Proposition 1, for \( k = L, H, HL \),

(30) \[ \frac{\partial p^k}{\partial G_1} = \frac{\partial p^k}{\partial E} \frac{\partial E}{\partial G_1} = \frac{\partial p^k}{\partial E} \theta_2 \frac{\partial G_2^k}{\partial G_1} \]

and

(31) \[ \frac{\partial p^k}{\partial D} = \frac{\partial p^k}{\partial E} \frac{\partial E}{\partial D} = \frac{\partial p^k}{\partial E} \left( 1 + \theta_2 \frac{\partial G_2^k}{\partial D} \right) \]

where \( \theta_2(\partial G_2^k/\partial G_1) \) and \( 1 + \theta_2(\partial G_2^k/\partial D) \) denote, respectively, the impact on public expenditures in period 2 of an additional unit of \( G_1 \) or \( D \). It follows that the relevant properties of the capitalization function depend on how \( G_2^k(\cdot) \) changes in response to the actions of the first generation. A similar remark applies for the capitalization function of the mixed institution.

A. Land-Tax-Only Institution

The following result characterizes the equilibria of the land-tax-only institution. Recall that \( G_2^k(G_1^k, D^k) \) denotes the equilibrium level of expenditures in IPGs by generation 2.

PROPOSITION 2: The land-tax-only institution generates equilibria with the following properties:

(i) Any feasible level of debt is an equilibrium, but it generates no intergenerational redistribution;
The level of expenditures in IPGs is Pareto optimal whenever $G^2_1(G_1^L, D^L) > 0$;

(iii) The level of IPG expenditures of the first generation is generically Pareto inefficiently low whenever $G^2_1(G_1^L, D^L) = 0$.

The intuition for this result follows directly from the following two properties of the capitalization function which are proved in the Appendix: $p^L(\cdot)$ is differentiable almost everywhere and satisfies

$$\frac{\partial p^L}{\partial D} = -1 \quad \text{and} \quad \frac{\partial p^L}{\partial G_1} = \begin{cases} \theta_2 \delta & \text{if } G_1^2(G_1, D) > 0 \\ 0 & \text{if } G_1^2(G_1, D) = 0. \end{cases}$$

The first property implies that any increase in the debt is fully offset by a decrease in land prices. As a result, any level of debt can be an equilibrium. Changes in debt levels have no impact on the allocation since the net transfer between the two generations remains constant.

A comparison of (32) and (29) shows that the necessary conditions for Pareto optimality are satisfied when the second generation IPG expenditures are positive. As a result, the level of IPGs chosen by the first generation is Pareto optimal when $G^2_1(G_1, D) > 0$.

By contrast, the necessary conditions are not satisfied when the second generation does not spend anything on the IPGs. In this case there is zero capitalization even though generation 2 still benefits from additional investments in period 1. This leads to an inefficient choice by the first generation who, in the absence of capitalization, ignore the positive impact of its choice on future generations.

It follows from the result that this institution generates optimal investment in economies where the rate of depreciation is sufficiently high ($\delta$ low), or when there is sufficient technological improvement ($\theta_1 > \theta_2$), since in both cases it is optimal for both generations to invest in the IPG. By contrast, the institution cannot induce optimal investment when $\delta \theta_2 > \theta_1$ (i.e., when it is cheaper to produce the IPGs in the first period), since in this case it is optimal for the first generation to be the sole provider of IPGs.

A more complete intuition for the result requires understanding why is it that the capitalization function takes the necessary form. By Proposition 1, we know that all future public expenditures are capitalized, i.e.,

$$(33) \quad p^L(G_1, D) = \delta - (D + \theta_2 G_1^2(G_1, D)).$$

In the Appendix we show that

$$(34) \quad G_1^2(G_1, D) = \max\{\bar{G}_2 - \delta G_1, 0\}$$

where $\bar{G}_2$ is the level of IPGs that generation 2 would purchase for itself if it had received no debt and no IPGs from the first generation. It follows that

$$(35) \quad p^L(G_1, D) = \delta - (D + \theta_2 \max\{\bar{G}_2 - \delta G_1, 0\})$$

which satisfies the properties described in (32). Note that the essential properties of the capitalization function depend on two key features of $G_1^2(G_1, D)$. First, IPG expenditures in period 2 are unaffected by the level of debt. Second, generation 2 always tries to consume $G_2$ units of the IPG. If $\delta G_1 = \bar{G}_2$, it invests nothing. If $\delta G_1 < \bar{G}_2$, it spends the amount necessary to reach the target level $\bar{G}_2$. In other words, the actions of the first generation do not affect generation 2’s desired consumption of IPGs. This is a direct consequence of the fact that, with full capitalization of public expenditures, there are no income effects: with land taxes the actions of the first generation do not change the wealth of the second generation.

Part (iii) of the result states that when $G_1^2(G_1^L, D^L) = 0$ Pareto inefficiency occurs generically, but not always. This qualifier is needed because of a minor technical complication that is described in detail in the proof. When the parameters of the economy are such that $G_1^2(G_1^L, D^L) = 0$ and $G_1 = \bar{G}_2 / \delta$, the price function is not
differentiable in equilibrium and some additional arguments are needed. Even in this case, however, the choice of generation 1 is inefficient except for a knife-edge subset of parameters.

Note that in this institution land prices can be negative. As can be seen from (35), land prices take their lowest value at \((0, D^{\max})\). If \(\tilde{p} < D^{\max} + \theta_2 G_2\), prices are negative in that state. By contrast, negative prices do not arise (in or out of equilibrium) when the baseline value of the land, \(\tilde{p}\), is sufficiently large relative to the debt ceiling and the level of public expenditures. Thus, the possibility of negative prices depends on the concrete parameters of the model. Negative prices are a natural feature of land taxation. Agents agree to sell at negative prices because otherwise they would be responsible for the taxes associated with the land.

B. Head-Tax-Only Institution

The following result characterizes the equilibrium of the head-tax-only institution:

**PROPOSITION 3:** The head-tax-only institution generates equilibria with the following properties:

(i) Intergenerational redistribution using the debt is possible and \(D^H = D^{\max}\);

(ii) The level of IPG expenditures of the first generation is always Pareto inefficiently low.

A comparison with Proposition 2 shows that a move from a land-tax-only to a head-tax-only institution has a significant impact on intergenerational exchange. Intergenerational expropriation through the debt is possible with head taxes, but not with land taxes. Present generations take advantage of this and raise the maximum amount of debt. Also, with head taxes current generations always underinvest in IPGs, whereas Pareto optimal investment is possible in the land-tax-only case.

An easy way to see the intuition behind this result is to consider the case of quasi-linear preferences, where \(u(c) = c\). As long as the endowment \(w\) is large enough to support interior allocations, the price of land in period 2 is fixed. Changes in \(G_1\) or \(D\) affect the well-being of the second generation, but not the price that they pay for land. As a result, the first generation fully ignores the intergenerational spillovers. It sets \(D^H = D^{\max}\). and chooses the level of \(G_1\) that it would have chosen in generational autarchy, which is inefficiently low.

The general intuition for this result follows directly from the following two properties of the capitalization function which are proved in the Appendix: \(p^H(\cdot)\) is differentiable almost everywhere and satisfies

\[
\frac{\partial p^H}{\partial D} \in (-1, 0]
\]

and

\[
\frac{\partial p^H}{\partial G_1} \in \begin{cases} [0, \theta_2 \delta] & \text{if } G^H_2(G_1, D) > 0 \\ \{0\} & \text{if } G^H_2(G_1, D) = 0. \end{cases}
\]

The first property implies that each additional unit of debt decreases land values, but by less than one unit. In this case, the net transfer from generation 2 to generation 1 increases with the size of the debt. It follows that the first generation sets \(D^H = D^{\max}\). The second property shows that the spillovers generated by \(G_1\) are always undercapitalized. If \(G^H_2(G_1, D) > 0\), each additional investment in period 1 generates an additional benefit of \(\theta_2 \delta\) for the second generation, but only part of this benefit is capitalized. As a result, the first generation does not fully internalize the spillover and underinvests in the IPG. Their incentive to invest is even lower when \(G^H_2(G_1, D) = 0\), since in that case there is zero capitalization.

Again, a more complete intuition for the result requires understanding why the capitalization function takes this form. As discussed above, by Proposition 1 we know that

\[
\frac{\partial p^H}{\partial D} = \frac{\partial p^H}{\partial E} \left( 1 + \theta_2 \frac{\partial G^H_2}{\partial D} \right)
\]

and

\[
\frac{\partial p^H}{\partial G_1} = \frac{\partial p^H}{\partial E} \frac{\partial G^H_2}{\partial G_1} \quad \frac{\partial G^H_2}{\partial G_1}.
\]

We also know that future public expenditures are undercapitalized, i.e., \(\partial p^H/\partial E \in (-1, 0]\).
The properties of capitalization function described in (36) then follow immediately from the following two properties of the policy response function proven in the Appendix:

\[
\frac{\partial G^H_2}{\partial D} \in \left( -\frac{1}{\theta_2}, 0 \right)
\]

and \( \frac{\partial G^H_2}{\partial G_1} \in (-\delta, 0] \).

These two properties are a direct consequence of the fact that, in contrast to the case of land taxes, the actions of generation 1 now have income effects. With partial capitalization, every additional unit of debt decreases the wealth of the second generation, and thus its expenditures in IPGs. Similarly, given the partial capitalization, as long as \( G^H_2(\cdot) > 0 \) every additional unit of IPGs increases the “full income” of the second generation, and thus its demand for IPGs.

In a nutshell, the differences between the two institutions can be traced back to the channels through which they operate. Land taxes operate solely through price effects. As a result, public expenditures in period 2 are fully capitalized and the actions of the first generation do not affect the wealth of the second. By contrast, head taxes operate solely through income effects. Therefore, only a fraction of the public expenditures in period 2 are capitalized, and the actions of the first generation affect the wealth of the second generation.

A natural question is whether the first generation purchases more IPGs in the land-tax-only institution. This is not the case in general. With head taxes, generation 1 internalizes only part of the spillovers, which reduces its demand for IPGs, but it also has more wealth since it can expropriate future generations, which increases its taste for all public goods. By contrast, with land taxes generation 1 internalizes more of the spillovers, but has less wealth. The relative importance of the two effects depends on the parameters of the model. For example, \( G^H_1 \geq G^H_2 \) in economies in which the debt ceiling is sufficiently low (\( D_{\text{max}} \approx 0 \)).

In contrast to the case of land taxation, in the Appendix we show that land prices are always positive in this institution.

C. Head-or-Land-Tax Institution

Now consider the head-or-land-tax institution where, in addition to voting over IPGs and debt, every generation \( t \) also chooses the fraction of total public expenditures (denoted by \( \phi_t \)) that is to be financed with land taxes.

We begin the analysis with a straightforward observation. Since each generation is homogeneous, every voter owns \( 1/N \)-th of the land, and thus pays \( 1/N \)-th of the taxes regardless of the tax base. It follows that any tax allocation \( \phi_t \in [0, 1] \) can be an equilibrium in every period and state of the world. This generates a continuum of equilibria.

Although any function \( \phi_2(G_1, D) \) can be a political equilibrium in period 2, for the purposes of this paper it is useful to restrict attention to a subset of equilibria in which the tax share is constant across states. The following result characterizes the equilibrium set under such a refinement:

PROPOSITION 4: The head-tax-or-land-taxes institution generates a continuum of equilibria with the following properties:

(i) Any endogenous choice of the tax base \((\phi_1, \phi_2) \in [0, 1]^2\) can arise as an equilibrium;

(ii) If \( \phi_2 = 1 \) (i.e., only land taxes are used in period 2) then the equilibrium allocation is identical to the one generated by the land-tax-only institution;

(iii) If \( \phi_2 < 1 \) (i.e., some head taxes are used in period 2) then \( D^{th} = D^{max} \) and the level of IPG investment by generation 1 is Pareto inefficiently low.

As we have seen, the choice of the tax base has a powerful effect on the extent to which intergenerational spillovers are capitalized, and thus on the behavior of the first generation. At the time generation 2 chooses its tax base, however, the actions of generation 1 and the capitalization effects have already taken place. As a result, generation 2 does not care about how its choice of tax instruments changes intergenerational incentives. Instead, its decision is based solely on intragenerational redistributional considerations. Since in a model with
homogenous voters redistribution through the choice of tax instruments is impossible, any split between the head and land tax bases can be an equilibrium.

There is an equilibrium in which generation 2 uses only land taxes that replicate the outcomes of the land-tax-only institution. But there is also a continuum of other equilibria in which only a fraction of expenditures are financed with land taxes. As long as \( \phi_2 < 1 \), public expenditures in period 2 are undercapitalized, which leads to a Pareto inefficiently low level of IPG investment by the first generation, and to the maximum level of debt. As the equilibrium tax base in period 2 moves from pure head taxes to pure land taxes, the level of IPG expenditures in period 1 goes from \( G_1^H \) to \( G_1^L \).

The main insight provided by Proposition 4 is that, although it would be extremely valuable for generation 2 to commit to use only land taxes in period 2, it cannot credibly do so. Generation 1 knows that the choice of tax base in period 2 will be based only on intragenerational redistributional considerations, and that this may lead to the use of head taxes.

This point is best made by introducing heterogeneity into the model in the simplest way possible. Suppose that every generation has two classes: the poor with endowment \( w_p \), and the rich with endowment \( w_R > w_p \). Assume that the poor constitute at least half of the population, so that the outcome of the election is the favorite policy of a representative member of the poor. Let \( l^P \) and \( l^R \) denote the land holdings in both periods. Consider the choice of the tax base under these circumstances. If head taxes are used, the poor “median voter” pays \( 1/N \)-th of the taxes. If land taxes are used, he pays a fraction \( l^P \) of the taxes (recall that there is one unit of land in the economy).

As a result, land taxes are chosen if the poor “median voter” owns less than \( 1/N \)-th of the land, and head taxes are chosen otherwise. In either case, the choice of the tax base is pinned down uniquely based on the redistributional forces in period 2, and not based on how its decision affects the incentives of previous generations. Interestingly, land taxes are rarely used by central governments. This suggests that the redistributitional politics at work in real economies select equilibria in which a small fraction of revenue, if any at all, is raised through land taxes. Proposition 4 shows that this decreases the incentives of current voters to internalize the impact of their policies on future generations.

We conclude this section with a comparison of the head-or-land-tax and the head-tax-only regimes. Although when \( \phi_2 \in (0, 1) \) both institutions generate maximal expropriation and inefficiently low investment in IPGs, the institutions are not equivalent. In particular, as shown in Proposition 1, the head-or-land-tax institution generates more capitalization of the IPG spillovers. It follows that, in general, \( G_{11}^{HL} \geq G_{11}^H \).

D. Mixed Institution

Finally consider the mixed institution. In this case each generation chooses the fraction \( \mu_2 \) of the debt that is financed with land taxes. By construction, all IPGs must be paid for with land taxes.

As before, for the purposes of this paper, it is useful to restrict attention to a subset of equilibria in which the fraction of debt expenditures in period 2 that is financed with land taxes is constant across states. The following result characterizes the equilibrium set under such a refinement:

**PROPOSITION 5:** The mixed institution generates a continuum of equilibria with the following properties:

(i) Any endogenous choice of the tax base \( (\mu_1, \mu_2) \in [0, 1]^2 \) can arise as an equilibrium;

15 The same logic extends more general choices among tax bases. Consider, for example, a proportional wealth tax. The poor favor a wealth tax over a land tax if and only if their share of the wealth is smaller than their share of the land.
(ii) If \( \mu_2 = 1 \) the equilibrium allocation is identical to the one generated by the land-tax-only institution;

(iii) If \( \mu_2 < 1 \) then \( D^M = D^{\max} \). In addition, if \( G^M_2(G^1_1, D^M) > 0 \) the level of IPG investment by generation 1 is Pareto optimal, and if \( G^M_2(G^1_1, D^M) = 0 \) it is inefficiently low.

Proposition 5 shows that the mixed institution shares with the land-tax-only institution its ability to induce optimal investment in IPGs (as long as \( G^M_2(G^1_1, D^{\max}) > 0 \)), but not its ability to stop intergenerational expropriation through the debt. As we will see in the next section, this will play a crucial role in the political feasibility of reform.

The properties of the mixed institution are a combination of those of the head-or-land-tax and land-tax-only institutions. Since no tax base restrictions are placed on how the debt is financed, it is undercapitalized as long as some head taxes are used in period 2. As a result, intergenerational redistribution is possible and the first generation selects the maximum level of debt. Furthermore, since all the IPG expenditures are financed with land taxes, there is full capitalization of those spillovers exactly as in the case of the land tax. For the same reasons as before, as long as \( G^M_2(G^1_1, D^M) > 0 \), this capitalization effect induces optimal investment by current generations.

E. Discussion

A comparison of the outcomes generated by the four institutions generates the following two insights.

The first insight is about how the well-known incidence properties of different tax bases affect the political economy of intergenerational expropriation and investment. When present generations believe that future generations will repay part of the debt with head taxes, which in the context of the model can be interpreted as a combination of non-land taxes, they know that the debt will be less-than-fully capitalized. This makes intergenerational redistribution through the debt possible and leads them to expropriate as much as possible.

The tax base restrictions also affect the incentives of the first generation to invest in IPGs. When present generations believe that future generations will finance future IPG expenditures with land taxes, even if there is no constitutional restriction requiring that this be the case, they know that these expenditures will be fully capitalized into land values. In many cases of interest, this gives them an incentive to invest optimally in a large class of IPGs. By contrast, if present generations believe that future generations will finance part of the future IPG expenditures with head taxes, they know that future public expenditures will be less-than-fully capitalized, and as a result they have insufficient incentives to invest optimally in IPGs.

The second insight is about the value, from an intergenerational point of view, of introducing constitutional restrictions on the tax base. The head-or-land-tax institution, with its absence of a tax-base restriction, is a good representation of the institutional status quo. In that institution the choice of tax instruments is driven by intragenerational redistributional politics, not by the objective of providing good intergenerational incentives. As a result, the politics may lead to non-land taxes, with its consequent loss in intergenerational incentives. Since in reality land taxes are rarely chosen by national governments, this seems to be the empirically relevant case.

Propositions 2 and 4 show that the introduction of a constitutional restriction amendment requiring IPGs and debt to be financed with land taxes makes intergenerational expropriation impossible and increases the incentives of present generations to invest in IPGs. Propositions 4 and 5 show that a weaker constitutional amendment, requiring that IPGs be financed with land taxes, but imposing no restrictions on how to finance the
debt, has a positive impact on IPGs, but not on expropriation. Finally, Propositions 3 and 4 show that a constitutional amendment restricting the use of land taxes, similar in spirit to California’s Proposition 13, reduces the incentives of the current generations to invest in IPGs, but has no impact on the debt.

Finally, consider a larger class of tax-base restrictions requiring that a minimum percentage of IPGs and/or debt expenditures be financed with land taxes. If the required percentage is less than 100, an extension of our results shows that such institutions are unable to induce optimal investment in IPGs and still allow for intergenerational expropriation. Nevertheless, reforms that increase the fraction of expenditures that are financed with land taxes have a positive impact on the incentives of present generations to invest in IPGs, and increase the extent to which land prices fall with the size of the debt.

IV. The Political Economy of Constitutional Change

In this section we study the political economy of constitutional reform. We assume that the economy is initially in the head-or-land-taxes regime, and in an equilibrium in which some head taxes are used, which seems to be a plausible representation of the institutional status quo. We then consider the political feasibility of two reforms: a move to a land-tax-only institution, and a move to a mixed institution. The case of a move to a head-tax-only system is not considered since, as we have shown, that institution does not have attractive intergenerational properties.

In our model the politics of reform are simple: the first generation supports a constitutional change if and only if it increases its welfare.

A. Politically Unfeasible Constitutional Change

The following result shows that a movement to land-tax-only institution is either politically unfeasible or neutral.\(^\text{16}\)

PROPOSITION 6: Suppose that some head taxes are used in period 2 in the head-or-land-tax institution. Then the welfare impact of moving from head-or-land-taxation to a land-taxation-only regime is as follows:

(i) If \(D_{\text{max}} = 0\) and \(G_2^{HL}(G_1^{HL}, D_{HL}) = 0\), the reform has no effect;
(ii) If \(D_{\text{max}} = 0\) and \(G_2^{HL}(G_1^{HL}, D_{HL}) > 0\), generation 1 is made worse off and generation 2 is made better off;
(iii) If \(D_{\text{max}} > 0\) generation 1 is made worse off. If in addition \(\theta_1 < \delta\theta_2\), then generation 2 is made better off.

Proposition 6 shows that the first generation opposes the reform since it never improves its welfare, and in most cases of interest, it decreases it. By contrast, except for the case \(\theta_1 < \delta\theta_2\) discussed below, generation 2 benefits from the reform whenever it is not neutral. Thus, although the reform increases the efficiency of the economy, it also entails a sizable transfer from present to future generations.

To see the intuition for the result, note that the reform has two effects. First, it protects future generations from expropriation through the debt. Second, it induces efficient investment in IPGs. Since the debt is purely a redistributive instrument, the first effect necessarily benefits future generations at the expense of current ones. It follows that the reform can generate a Pareto improvement only if the efficiency gains from the IPG investment are sufficiently large, and generation 1 receives a sufficiently large share of them.

Parts (i) and (ii) are crucial for understanding the result. They show that even when there is no debt, the first generation is (weakly) hurt by the reform. It follows that the first generation is hurt by the reform when \(D_{\text{max}} > 0\), since there are no gains from the IPG dimension that can be used to compensate it for losing the ability to expropriate.

To see why the reform cannot be Pareto improving even when \(D_{\text{max}} = 0\), consider the case of quasi-linear preferences, and assume that only head taxes are used in the head-or-land-tax institution. Figure 2 plots the equilibrium land prices for both institutions under these assumptions. Note several things. First, prices are responsive to \(G_1\) only in the land-tax-only institution. Second, the prices in the land-tax-only regime are

\(^{16}\) Recall that \(G_2^{HL}(G_1^{HL}, D_{HL})\) denotes the level of IPG expenditures that generation 2 makes in equilibrium.
never above those of the head-or-land-tax regime, and if \( G_1 < G_2 / \delta \), they are strictly below. In fact, if \( G_1 = 0 \), land values go down by the full cost of providing the IPGs in period 2. Third, as is shown in the Appendix, generation 2 invests a positive amount in IPGs if and only if \( G_1 < G_2 / \delta \).

This picture illustrates both the incentive and welfare effects of the reform. A move to land taxation increases the incentives of the first generation to invest in the IPGs since it is rewarded with higher land prices. However, the reform also shifts the incidence of providing IPGs in period 2 to the first generation. This incidence shift entails a sizable transfer from present to future generations. In fact, as the picture starkly illustrates, the capitalization benefits from investing more in the IPG allows the first generation to escape the incidence shift only if it invests more than \( G_2 / \delta \), which crowds out investment by the second generation. To see this point, compare the welfare of the first generation in the land-tax-only and head-or-land-tax institutions when it makes an identical choice \((G_1, D)\). Generation 1 is worse off in the land-tax-only institution as long as

\[
\begin{align*}
(39) \quad u(w - \frac{p^l(G_1, D) + G_1 - D}{N}) \\
\qquad \leq u(w - \frac{p^{HL}(G_1, D) + G_1 - D}{N}),
\end{align*}
\]

i.e., as long as \( p^l(G_1, D) \leq p^{HL}(G_1, D) \).

The picture also shows why the reform is neutral in some cases. When \( \delta G_1 > G_2 \), the capitalization function is identical for both institutions. If the parameters of the economy are such that the optimal choice of the first generation falls in this region, then there is no difference between the two institutions.

A perplexing feature of the result is that when \( \delta \theta_2 \) is sufficiently larger than \( \theta_1 \), and \( D_{\text{max}} > 0 \), the reform can decrease the welfare of both generations. The problem is that in this case there is a “money pump.” To see this, consider the following hypothetical scheme: reduce the IPG expenditures of generation 2 by one unit, and transfer that unit of the private good to generation 1. This decreases the amount of IPGs in period 2 by \( 1/\theta_2 \). But by increasing the wealth of generation 1 by one unit, it also leads to an increase in the amount of IPGs bequeathed to the second generation equal to

\[
(40) \quad \frac{\delta}{\theta_1} \frac{\partial G^{HL}_1}{\partial w}.
\]

The net effect is positive if \( \delta \theta_2 \) is sufficiently larger than \( \theta_1 \). When there is a money pump, generation 2 can favor the head-or-land-tax institution over the land-tax-only regime. In that case generation 2 knows that each unit of transfers to the first generation more than pays for itself. As a result, it wants to transfer as many resources as possible. The head-or-land-tax institution with a positive debt-ceiling allows for such transfers. The land-tax-only institution does not. As shown in the Appendix, this type of money pump is not possible when \( \theta_1 > \delta \theta_2 \).

B. Politically Feasible Constitutional Change

Now consider a move to a mixed institution that is accompanied by a sufficiently large increase in the debt ceiling. Suppose that the reform does not affect the fraction of expenditures in period 2 that is financed with land taxes whenever a choice between head and land taxes is available (i.e., \( \phi_2 = \mu_2 \)). The following result shows that the reform induces a Pareto improvement and thus is politically feasible.
PROPOSITION 7: Consider a move from a head-or-land-tax institution to a mixed institution that is accompanied by an increase in the debt ceiling of $dD^\max = \theta_2 G_{2}^{HL}(G_{1}^{HL}, D^\max)$. If the amount invested in IPGs by the first generation increases, and $D_{\max} + dD^\max < Nw$, the reform is feasible and Pareto improving.

There are two problems with the move to a land-tax-only system: generation 1 loses its ability to expropriate the second generation, and the incidence of providing IPGs in period 2 is shifted to present generations. The two components of the reform considered in Proposition 7 address these problems. First, by imposing a tax-base restriction for the financing of IPGs, but not for the debt, intergenerational expropriation is still possible. Second, by increasing the size of the debt ceiling, and thus the amount that can be expropriated, it compensates the first generation for the incidence shift. The result then follows since, as shown in the Appendix, an increase of the debt-ceiling equal to $\theta_2 G_{2}^{HL}(G_{1}^{HL}, D^\max)$ is feasible and sufficient to compensate the first generation without making the second generation worse off. Note that using a mixed institution is crucial. Increases in the debt-ceiling can be used to compensate the first generation only if they are not financed with land taxes.

C. Discussion

Propositions 6 and 7 show that there is a trade-off between political feasibility and the degree of protection that is given to future generations. A move to a land-tax-only institution is required to increase the incentives of present generations to invest in IPGs and to protect future generations from expropriation. As we have seen, this is politically unfeasible. By contrast, within the class of institutions studied in this paper, it is politically feasible to increase the incentives of present generations to invest in IPGs, but not to protect future generations from expropriation.

These insights about the political economy of fiscal assignment also apply to a more realistic model in which there are multiple public goods, some of which do not generate intergenerational spillovers, and in which the government has redistributive objectives, which may take the form of social insurance. In such a world, the fiscal assignment problem requires specifying a tax-base restriction for each type of expenditure. The results in this paper suggest two principles:

(a) Assigning IPGs to a land-tax base increases the incentives of present generations to invest in these programs, and in many cases of interest induces full internalization of the intergenerational spillovers; and

(b) Assigning debt financing to a land-tax base is sufficient to stop intergenerational expropriation through the debt.

Importantly, our results have nothing to say about the optimal fiscal assignment of intragenerational programs.

V. Limitations and Extensions

A. Elastic Land Supply and Alternative Tax Bases

As is well-known in tax incidence theory, a tax on the sale of an asset falls fully on the owners of the asset only if the supply is inelastic. Our results can be extended for the case of elastically supplied assets (e.g., when land improvements are possible), but they take the form of a limit result: as the elasticity of supply goes to zero, the outcomes generated by the institutions converge to those characterized above. With elastically supplied assets, Pareto optimal provision of IPGs is no longer possible in the land-tax-only and mixed institutions. As long as the supply of land is sufficiently inelastic, however, both institutions still increase the production of IPGs relative to the head-or-land-tax regime. Furthermore, if current generations can increase the supply of land, but not decrease it, the results on the debt still hold. Since increases in the debt reduce land prices, the capitalization effect could be undone only with a decrease in the land stock, which is not feasible.

The discussion also suggests that our results can be extended to any other assets that are sold from present to future generations; are sufficiently inelastically supplied; and have a
sufficiently large aggregate value. As shown at the end of section III A, the last condition is important to rule out negative prices.

B. Non-Additively Separable Preferences

We have assumed that preferences are additively separable, which simplifies the analysis by making the demand for land of the second generation independent of the level of IPGs that they consume. This assumption plays a role in the properties of the capitalization function derived in Proposition 1.

When land and IPGs are not additively separable (for example, if preferences take the form \( c + v(l, G) \)), the demand for land \( \lambda(\cdot) \) depends on \( G_2 \). This introduces an additional capitalization mechanism: changes in the amount of IPGs consumed by the second generation can change land prices even if public expenditures in period 2 remain constant.

For IPGs such as pure R&D, in which \( \partial \lambda / \partial G_2 \approx 0 \), this additional mechanism is negligible and our results still apply. For IPGs where \( \partial \lambda / \partial G_2 \) is large, it can be shown that the additional mechanism changes the performance of the institutions in which head taxes are used. Importantly, the additional capitalization effect can be positive (when land and the IPG are complements as in the case of roads) or negative (when they are substitutes as in the case of land and national parks), and is not directly related to the consumption value of the IPG. As a result, except for special cases, head taxation still leads to inefficient levels of IPGs and to maximal expropriation. By contrast, the additional mechanism does not interfere with the ability of land taxation to induce Pareto optimal investment in IPGs.

C. Reversible IPGs

We have also assumed that the IPG is non-reversible. The reversibility assumption is not important. In fact, a straightforward generalization of our arguments shows that one gets even stronger results for reversible IPGs: in the land-tax-only and mixed institutions, the level of IPGs is always Pareto optimal (instead of only when \( G^*_2(G_1, D) > 0 \), as is the case for non-reversible IPGs).

VI. Conclusions

This paper has studied how to protect future generations from expropriation and to induce optimal investment in IPGs by introducing constitutional restrictions on the tax base. We have shown that the introduction of a constitutional amendment requiring that IPGs and debt be financed with land taxes would make intergenerational expropriation impossible and, for many cases of interest, would induce optimal investment in IPGs. We have also shown that a weaker constitutional amendment requiring that IPGs be financed with land taxes, but imposing no restrictions on how to finance the debt, would have a positive impact on IPGs, but not on expropriation. The first reform is not politically feasible, since it hurts the first generation, but the second weaker reform can induce a Pareto improvement, and thus could be supported by the first generation.

We conclude the paper with several qualifications to our results. First, our case for a land-tax base has ignored the cost of administering the different tax bases and their intragenerational redistributional properties. With respect to the first, we have considered a tax _per-unit of land_, as opposed to a tax on land market values, since presumably the former is easier to administer. Still, administering a land tax might prove prohibitively difficult for some countries. With respect to the issue of intragenerational redistribution, land taxes might be more regressive than the mix of income and commodity taxes used in most countries. The value of improved intergenerational incentives needs to be weighted against these other dimensions.

Second, as discussed in the introduction and in section I D, tax-base restrictions are not the only mechanisms for providing intergenerational incentives. Clearly, a constitutional restriction to a land-tax base is not necessary in an economy where those other mechanisms are sufficient to stop expropriation and to induce optimal investment in IPGs. Since casual observation suggests that these other mechanisms are

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17 For a discussion of the administrative difficulties involved in land taxation, see Oates and Schwab (1997).
not able to do this, however, there seems to be a role for providing additional incentives through tax-base restrictions.

Third, introducing a restriction to a land-tax base can help with the provision of IPGs but cannot take care of other types of public inter-generational spillovers. As we have shown, land taxes capitalize the impact of current policy on future public expenditures, not the value that future generations place on current policy choices. As a result, tax base restrictions cannot be used to induce an optimal choice of policies that have little or no effect on future public expenditures. For example, consider a hypothetical IPG that has the following features: it can be provided only by current generations; it is very valuable for future generations; and it has no effect on the behavior of future generations.\footnote{For example, because it enters as an additively separable term in their utility function.} Regardless of how valuable it is, such a “pure IPG” will not be provided by the institutions studied here.\footnote{See Rangel (2003) for an explanation of how “political capitalization” can induce present generations to provide this type of intergenerational good.} Another example is the choice of public regulations regarding the behavior of charitable trust and endowment institutions.

Finally, our results have been derived in a stark two-period model with homogeneous jurisdictions. Although the intuitions developed here suggest that the results should extend to a general OLG model with multiple overlaps, it would be interesting to demonstrate formally that this is indeed the case.

APPENDIX

The following properties are used in the proofs.

PROPERTY 1: For all $x > 0$ there exists a unique land price $\pi(x)$ for which $N. \lambda(\pi(x)|x) = 1$.

PROOF OF PROPERTY 1:

The assumptions on preferences imply that, for all $x > 0$: (a) $\lambda(p|x)$ is a strictly positive and continuously differentiable function; (b) $\lambda(p|x) > 1/N$ for $p$ sufficiently low; and (c) $\lambda(p|x) \to 0$ as $p \to \infty$. These three properties imply that there exists $p$ such that $N\lambda(p|x) = 1$. Uniqueness follows from the fact that the FOC $pu'(x - plN) = v'(1/N)$ must be satisfied at $\pi(x)$, and the left-hand side of this condition is increasing in $p$.

PROPERTY 2: For all $x > 0$, $(1/N) (\partial \pi(x)/\partial x) \in [0, 1]$.

PROOF OF PROPERTY 2:

Since $\pi(x)$ is implicitly defined by $N \lambda(\pi(x)|x) = 1$, the Implicit Function Theorem (IFT) implies that $\partial \pi/\partial x = -\lambda_p/\lambda_x \geq 0.\footnote{\lambda_p$ and $\lambda_x$ denote, respectively, the derivative of the land demand function with respect to prices and wealth.}$ Furthermore, since $\lambda(p|x)$ is defined implicitly by $pu'(x - pl) = v'(l)$, the IFT implies that $\lambda_p/\lambda_x(\pi(x), x) = -\lambda(\pi(x)|x) + u'/pu'' < -\lambda(\pi(x)|x)$. It follows that $\lambda(\pi(x)|x)\partial \pi/\partial x < 1$. Property 2 follows since, by definition, $\lambda(\pi(x)|x) = 1/N$.\footnote{For example, because it enters as an additively separable term in their utility function.}
The following function is used in some of the proofs below. Let

\[ G^*_2(G_1, D, p) \equiv \arg \max_{G \geq 0} \left( w - \frac{p}{N} - \frac{D + \theta_2 G}{N} \right) + g(\delta G_1 + G_2) \]

denote the level of IPG expenditures that the representative voter of generation 2 chooses as a function of the state and the land price \( p \). Note that, since all of the members of a generation are identical, they always pay \( 1/N \)-th of the taxes, and thus \( G_2^*(\cdot) \) characterizes the policy problem of generation 2 regardless of the tax base. By the assumptions on preferences, \( G_2^*(G_1, D, p) \) is a uniquely defined function with the following properties:

(a) There exists \( \hat{p}(G_1, D) \leq \text{Nw} - D \) such that \( G_2^*(G_1, D, p) = 0 \) for all \( p \geq \hat{p}(G_1, D) \), and \( G_2^*(G_1, D, p) > 0 \) for all \( p < \hat{p}(G_1, D) \);

(b) \( G_2^*(G_1, D, p) \) is continuously differentiable for \( p < \hat{p}(G_1, D) \); and \( \theta_2 \partial G_2^*/\partial p \in (-1, 0] \);

(c) \( \theta_2 G_2^*(G_1, D, 0) < \text{Nw} - D \).

**PROOF OF PROPOSITION 1:**

(Step 1) Let \( \phi^k \) denote the fraction of expenditures financed with land taxes in institution \( k = L, H, HL \) (with \( \phi^L = 1, \phi^H = 0 \), and \( \phi^{HL} \in [0, 1] \)). For any level of expenditures \( E \) in period 2, the land market clearing condition in institution \( k \) is given by

\[ N\lambda \left( p^k(G_1, D) + \phi^k E \right) \left( w - \frac{(1 - \phi^k)E}{N} \right) = 1. \]

By Property 1, the equilibrium price is given by

\[ p^k(G_1, D) = \pi \left( w - \frac{(1 - \phi^k)E}{N} \right) - \phi^k E. \]

By Property 2, it follows that

\[ \frac{\partial p^k}{\partial E} = - \frac{(1 - \phi^k)}{N} \frac{\partial \pi}{\partial x} - \phi^k \in (-1, -\phi^k], \]

where \((-1, -1] \) is defined to be equal to \(-1 \).

(Step 2) Now consider the mixed institution. The land market clearing condition is given by

\[ N\lambda \left( p + \mu_2 D + \theta_2 G_2 \right) \left( w - \left(1 - \mu_2 \right) \frac{D}{N} \right) = 1. \]

From Property 1 it follows that

\[ p^M(G_1, D) = \pi \left( w - \left(1 - \mu_2 \right) \frac{D}{N} \right) - (\mu_2 D + \theta_2 G_2). \]

The properties of \( \partial p^M/\partial G_2 \) and \( \partial p^M/\partial D \) then follow directly from Property 2.
PROOF OF PROPOSITION 2:

(Step 1) We begin by showing that, for every state \((G_1, D)\), there exists a unique pair \((p_L(G_1, D), G_2^*)\) that satisfies the conditions for land market and political equilibrium in period 2. We refer to such a pair as a “continuation equilibrium.” The set of all continuation equilibria can be characterized as the intersection of the following two loci in \((p, \theta_2 G_2)\)-space:

(a) The locus \((p, \theta_2 G_2(G_1, D, p))\) that pairs each possible price \(p\) with the unique political equilibrium consistent with that price;

(b) The locus \((\bar{p} - (D + \theta_2 G_2), \theta_2, G_2)\) that, by Proposition 1, pairs each possible \(\theta_2 G_2\) with the unique equilibrium land price that is consistent with that anticipated level of IPG expenditures.

The two loci are plotted in Figure 3. The properties of locus 1 are described above. Locus 2 satisfies the following properties: (a) it intersects the \(p\)-axis at \(\bar{p} - D\), and (b) it decreases linearly with slope \(-1\) for any \(p < \bar{p} - D\). Since all the goods are normal, generation 2 spends only a fraction of its wealth in land. It follows that \(\bar{p} - D < \bar{N}w - D\) and thus the two loci intersect exactly once in either the first or the second quadrants (without more restrictions, the land prices cannot be guaranteed to be positive). The existence, uniqueness, and continuity of the continuation mapping follows.

(Step 2) We claim that

\[
G_2^*(G_1, D) = \max\{\bar{G}_2 - \delta G_1, 0\}
\]

and

\[
p_L(G_1, D) = \bar{p} - (D + \theta_2 G_2^*(G_1, D))
\]

where \(\bar{G}_2\) is defined implicitly by

\[
\theta_2 = N \frac{g'((\bar{G}_2))}{u'(w - \frac{\bar{p}}{N})}.
\]

(Note that \(\bar{G}_2\) is the level of IPGs that generation 2 would choose in state \((0, 0)\)). The shape of \(p_L(G_1, D)\),
D) follows from Proposition 1. Now consider the policy problem of generation 2 which, given the form of the capitalization function, can be written as

\[
A_{10} \quad \max_{G_2 \geq 0} u \left( w - \frac{\bar{p}}{N} + \theta_2 \frac{G_2(G_1, D) - G_2}{N} \right) + g(\delta G_1 + G_2).
\]

Since \(G_2^*(G_1, D)\) is, by definition, the unique solution to the problem, the following (necessary and sufficient) first-order conditions must be satisfied:

\[
A_{11} \quad \theta_2 \geq N \frac{g'(\delta G_1 + G_2)}{u'(w - \bar{p}/N)}, \text{ with equality if } G_2 > 0.
\]

It follows that \(G_2^*(G_1, D) = \max\{G_2 - \delta G_1, 0\}\).

(Step 3) Consider the policy choice of generation 1 which, substituting (A7) and (A8) into (25), can be written as

\[
A_{12} \quad \max_{G_1 \geq 0, D \in \text{pass}} u \left( w - \frac{\theta_1 G_1}{N} + \frac{\bar{p} - \theta_2 \max\{G_2 - \delta G_1, 0\}}{N} \right) + f(G_1).
\]

Since the objective function is independent of \(D\), any level of debt can be an equilibrium. Furthermore, since land consumption, the amount that generation 2 transfers to generation 1 (equal to \(p^*(G_1, D) + D = \bar{p} - \theta_2 \max\{G_2 - \delta G_1, 0\}\)), and generation 2’s expenditure and total consumption of IPGs are all independent of \(D\), it follows that the allocation is independent of \(D\).

(Step 4) A comparison of (A11) and (7) shows that \(G_2^*(G_1, D)\) satisfies the Samuelson condition for \(G_2\). By the maximization problem of generation 1 described in (A12), if \(G_2^*(G_1^*, D) > 0\), the choice of generation 1 must satisfy the FOC

\[
A_{13} \quad u'(\bar{w} + \theta_1 G_1/N + \theta_2 \delta) + f' = 0.
\]

Since this condition is equivalent to the Samuelson condition for \(G_1\) described in (9), it follows that all of the necessary and sufficient conditions for Pareto optimality are satisfied when \(G_2^*(G_1^*, D) > 0\).

(Step 5) Finally, suppose that \(G_2^*(G_1^*, D) = 0\). There are two cases to consider.

(case i) \(\delta G_1^* > G_2\). By the maximization problem of generation 1 described in (A12), the choice of generation 1 must satisfy the FOC

\[
A_{14} \quad N \frac{f'}{u_i} = \theta_1.
\]

Consider forcing generation 1 to increase its production of the IPG by a marginal amount. By (A14), this has no effect on generation 1’s welfare, but generates a positive benefit for generation 2. Since generation 2’s consumption is positive in every state, it follows that a Pareto improvement is possible.

(case ii) \(\delta G_1^* = G_2\). In order for this to be an optimal choice for generation 1 it must be the case that
(A15) \[ Nf' - \theta_1 u'_1 \leq 0 \quad \text{and} \quad -Nf' + (\theta_1 - \theta_2)u'_1 \leq 0, \]
i.e., it cannot gain from increasing or decreasing \( G_1 \) given the capitalization function. It follows that the following FOC must be satisfied:

(A16) \[ Nf' \frac{u'}{u_1'} + \mu = \theta_1, \text{for some } \mu \in \begin{cases} [0, \delta \theta_2] & \text{if } \theta_1 > \delta \theta_2 \\ [0, \theta_1] & \text{if } \theta_1 \leq \delta \theta_2. \end{cases} \]

Note that \( \delta G_1^f = \bar{G}_2 \) implies that \( N(g'/u_2) = \theta_2 \). Thus, if \( \mu = \delta \theta_2 \), the FOC is equivalent to the Samuelson condition for \( G_1 \) described in (8), and the allocation is Pareto optimal. If \( \mu < \delta \theta_2 \), the level of \( G_1 \) is Pareto suboptimally low. To see this, consider forcing generation 1 to increase its production of the IPG by a marginal amount. By (A16), generation 1 requires a transfer of \( \mu \) to remain indifferent. But the additional unit generates a marginal benefit for generation 2 equal to \( \delta \theta_2 \). It follows that a Pareto improvement is possible since the benefits to generation 2 exceed the required compensation to generation 1.

To conclude the proof, note that the subset of parameters for which \( \delta G_1^f = \bar{G}_2 \) and \( \mu = \delta \theta_2 \) has measure zero in the set of all possible economies satisfying the assumptions of the model.

PROOF OF PROPOSITION 3:

(Step 1) We begin by showing that, for every state \((G_1, D)\), there exists a unique pair \((p^H(G_1, D), G_2^H(G_1, D))\) that satisfies the conditions for land market and political equilibrium in period 2. We refer to such a pair as “continuation equilibrium.” The set of all continuation equilibria can be characterized as the intersection of the following two loci in \((p, \theta_2 G_2)\)-space:

(a) The locus \((p, \theta_2 G_2(G_1, D, p))\) that pairs each possible price \( p \) with the unique political equilibrium consistent with that price;
(b) The locus \((\pi(w - ((D + \theta_2 G_2)/N)), \theta_2 G_2)\) that, by Property 1, pairs each possible \( \theta_2 G_2 \) with the unique equilibrium land price that is consistent with that anticipated level of IPG expenditures.

The two loci are plotted in Figure 4. Locus 2 satisfies the following properties: (a) it intersects the \( p \)-axis at \( \pi(w - D/N) \); and (b) it lies to the right of the \( \theta_2 G_2 \)-axis whenever \( \theta_2 G_2 < Nw - D \). Note also that, by the properties of \( G_2^H(\cdot) \) listed above, the intersection of the first locus with the \( \theta_2 G_2 \) is below \( Nw - D \). Since all the goods are normal, generation 2 spends only a fraction
of its wealth in land. It follows that $\pi(w - D/N) < Nw - D$ and thus the two loci intersect exactly once in the first quadrant. The existence, uniqueness, and continuity of the continuation equilibria mapping follows.

(Step 2) $G^H_2(G_1, D)$ is implicitly defined by the FOC

$$\frac{\theta_2}{N} u' \left( \frac{w - \frac{D + \theta_2 G_2}{N}}{N} - \frac{D + \theta_2 G_2}{N} \right) = g'(\delta G_1 + G_2) \tag{A17}$$

with equality if $G_2 > 0$. (Note that we have used the fact that, by Property 1, the equilibrium price is given $\pi(w - (D + \theta_2 G_2)/N)$.

For all $D$, let $\hat{G}^H_1(D)$ be the level of $G_1$ at which

$$\frac{\theta_2}{N} u' \left( \frac{w - \frac{D}{N}}{N} - \frac{D}{N} \right) = g'(\delta \hat{G}^H_1(D)). \tag{A18}$$

Note that, by construction, $G^H_2(G_1, D) = 0$ for all $G_1 \geq \hat{G}^H_1(D)$. We now characterize some key properties of $G^H_2(G_1, D)$. There are three cases to consider.

(case i) $G^H_2(G_1, D) > 0$. In this case, the IFT implies that $G^H_2(\cdot)$ is differentiable at $(G_1, D)$, with

$$\frac{\partial G^H_2}{\partial G_1} = -\frac{\delta g''}{g'' + \frac{\theta_2}{N} \left( 1 - \frac{1}{N} \frac{\partial \pi}{\partial x} \right) u''} \tag{A19}$$

and

$$\frac{\partial G^H_2}{\partial D} = -\frac{1}{g'' + \frac{\theta_2}{N} \left( 1 - \frac{1}{N} \frac{\partial \pi}{\partial x} \right) u''}. \tag{A20}$$

By Property 2, and the strict concavity of $g$ and $u$, we get that

$$\frac{\partial G^H_2}{\partial G_1} \in (-\delta, 0] \quad \text{and} \quad \frac{\partial G^H_2}{\partial D} \in \left(-\frac{1}{\theta_2}, 0\right]. \tag{A21}$$

(case ii) $G^H_2(G_1, D) = 0$ and $G_1 > \hat{G}^H_1(D)$. In this case (A17) holds with strict inequality and thus

$$\frac{\partial G^H_2}{\partial G_1} = 0 \quad \text{and} \quad \frac{\partial G^H_2}{\partial D} = 0. \tag{A22}$$

(case iii) $G^H_2(G_1, D) = 0$ and $G_1 = \hat{G}^H_1(D)$. In this case $G^H_2(\cdot)$ is not differentiable at $(G_1, D)$. 

\[ \text{Note:}\]
(Step 3) By Proposition 1, we know that

\( \frac{\partial p^H}{\partial G_1} = \frac{\partial p^H}{\partial E} \theta_2 \frac{\partial G_2^H}{\partial G_1} \quad \text{and} \quad \frac{\partial p^H}{\partial D} = \frac{\partial p^H}{\partial E} \left( 1 + \theta_2 \frac{\partial G_2^H}{\partial D} \right). \)

By Step 2, the capitalization function is differentiable almost everywhere. By Step 2, and Proposition 1, the derivative (whenever it exists) satisfies the following properties:

\( \frac{\partial p^H}{\partial G_1} \in \begin{cases} [0, \theta_2 \delta) & \text{if } G_2^H(G_1, D) > 0 \\ 0 & \text{if } G_2^H(G_1, D) = 0 \end{cases} \)

and

\( \frac{\partial p^H}{\partial D} \in (-1, 0]. \)

(Step 4) Finally, consider the policy problem of generation 1 described in (25). By (A25) and the continuity of the capitalization function we get that \( D = D_{\text{max}}. \) Now consider the choice of \( G_1. \) We need to consider three cases.

(case i) \( G_2^H(G_1, D) > 0. \) In this case, by Step 3, the choice of generation 1 must satisfy the FOC

\( N \frac{f'}{u'_1} + \mu = \theta_1, \quad \text{with } \mu \in [0, \theta_2 \delta). \)

Using an argument similar to the one in Step 5 of the previous proof, we get that \( G_1^H \) is Pareto inefficiently low.

(case ii) \( G_2^H(G_1, D) = 0 \) and \( G_1 > \hat{G}_1(D). \) In this case the choice of generation 1 must satisfy the FOC

\( N \frac{f'}{u'_1} = \theta_1 \)

which using similar arguments again implies that \( G_1^H \) is Pareto inefficiently low.

(case iii) \( G_2^H(G_1, D) = 0 \) and \( G_1 = \hat{G}_1(D). \) In order for this to be an optimal choice for generation 1 it must be the case that

\( N f' - \theta_1 u'_1 \leq 0 \quad \text{and} \quad -N f' + (\theta_1 - \gamma) u'_1 \leq 0, \) for some \( \gamma \in [0, \theta_2 \delta), \)

i.e., it cannot gain from increasing or decreasing \( G_1 \) given the capitalization function. It follows that the following FOC must be satisfied:

\( N \frac{f'}{u'_1} + \mu = \theta_1, \) for some \( \mu \in \begin{cases} [0, \gamma) & \text{if } \theta_1 > \delta \theta_2 \\ [0, \min(\gamma, \theta_1)] & \text{if } \theta_1 \leq \delta \theta_2 \end{cases}. \)

Once more, regardless of which case applies, a repetition of previous arguments shows that \( G_1^H \) is Pareto inefficiently low.
PROOF OF PROPOSITION 4:
Parts (i) and (ii) are simple and proven in the text. The proof of part (iii) is a straightforward extension of the previous proof. For this reason we sketch only the most salient parts of the argument. Suppose that $\phi_2 < 1$. In step 1 of the proof for Proposition 1 we showed that

\[ p^{HL}(G_1, D) = \pi \left( w - (1 - \phi_2) \frac{D + \theta_2 G^{HL}_{2}(G_1, D)}{N} \right) - \phi_2 (D + \theta_2 G^{HL}_{2}(G_1, D)). \]

It follows that

\[ \frac{\partial p^{HL}}{\partial D} = - \frac{(1 - \phi_2)}{N} \frac{\partial \pi}{\partial x} - \phi_2 - \theta_2 \frac{\partial G^{HL}_{2}}{\partial D} > -1 \]

where the inequality follows by Property 2 and the fact that $\partial G^{HL}_{2}/\partial D \leq 0$. Similarly, (A30) implies that

\[ \frac{\partial p^{HL}}{\partial G_1} = - \theta_2 \frac{\partial G^{HL}_{2}}{\partial G_1} \left[ \frac{(1 - \phi_2)}{N} \frac{\partial \pi}{\partial x} + \phi_2 \right]. \]

Since one can show that $\partial G^{HL}_{2}/\partial G_1 \in (-\delta, 0]$, it follows that

\[ \frac{\partial p^{HL}}{\partial G_1} \in \begin{cases} [0, \delta \theta_2] & \text{if } G^{HL}_{2}(G_1, D) > 0 \\ \{0\} & \text{otherwise} \end{cases}. \]

The properties of the policy choices in period 1 immediately follow.

PROOF OF PROPOSITION 5:
Since the proof of this result is a straightforward extension of the proofs of Propositions 2 and 3, we sketch only the most salient parts of the argument. In step 2 of the proof for Proposition 1 we showed that

\[ p^M(G_1, D) = \pi \left( w - (1 - \mu_2) \frac{D}{N} \right) - (\mu_2 D + \theta_2 G^{M}_{2}(G_1, D)). \]

For all $D$, define $G^M_{2}(D)$ to be the level of $G_2$ at which $G_{2}^M(G_1, D) = 0$ for all $\delta G_1 \geq G^M_{2}(D)$. Given this, one can show that

\[ G_{2}^M(G_1, D) = \max\{G^M_{2}(D) - \delta G_1, 0\}. \]

It follows that, whenever they are differentiable,

\[ \frac{\partial p^M}{\partial G_1} = \begin{cases} \theta_2 \delta & \text{if } G_{2}^M(G_1, D) > 0 \\ 0 & \text{if } G_{2}^M(G_1, D) = 0 \end{cases}, \]

and

\[ \frac{\partial p^M}{\partial D} = - \frac{(1 - \mu_2)}{N} \frac{\partial \pi}{\partial x} - \mu_2 - \theta_2 \frac{\partial G^M_2}{\partial D} > -1 \]

since $\partial G^M_2/\partial D \leq 0$. The properties of the policy choices in period 1 immediately follow.
PROOF OF PROPOSITION 6:
Let $U^k_t(G_1, D)$ denote level of welfare of generation $t$ in institution $k$ conditional on generation 1 choosing $(G_1, D)$. Let $U^k$ denote the equilibrium welfare levels. Let $T^k(G_1, D) = \delta G_1 + G^k_2(G_1, D)$ and $c^k_2(G_1, D) = w - (1/N) (p^k(G_1, D) + D + \theta c^k_2(G_1, D))$ be the amount of IPGs and private goods consumed by generation 2. Also, let $V^k_2(D^{max})$ denote the equilibrium payoff for generation 2 as a function of the debt ceiling.

(Step 1) We claim that for all $(G_1, D)$, $U^H(G_1, D) \equiv U^L(G_1, D)$, with equality if and only if $D = 0$ and $G_1 \geq \tilde{G}_2/\delta$. Note that

$$U^H_l(G_1, D) \equiv U^L_l(G_1, D) \text{ if and only if } p^H_l(G_1, D) \equiv p^L_l(G_1, D).$$

The claim then follows from Proposition 1 and the fact that $p^H_l(G_1, 0) = p^L_l(G_1, 0)$ for $G_1 \geq \tilde{G}_2/\delta$.

(Step 2) We claim that, when $D^{max} = 0$, $G^H_1 \leq G^L_1$, with equality if and only if $G^H_1 \geq \tilde{G}_2/\delta$. This follows directly from the properties of $p^H_l(\cdot, 0)$ and $p^L_l(\cdot, 0)$.

(Step 3) Consider first the impact of the reform on generation 1. There are three cases to consider:

(case i) Suppose that $D^{max} = 0$ and $G^H_2(G^H_1, 0) = 0$. It follows that $G^H_1 \geq \tilde{G}_2/\delta$ and, by Step 2, $G^H_1 = G^L_1$. By Step 1 it follows that $U^H_l = U^L_l$.

(case ii) Suppose that $D^{max} = 0$ and $G^H_2(G^H_1, 0) > 0$. It follows that $G^H_1 < \tilde{G}_2/\delta$ and, by Step 2, $G^H_1 < G^L_1$. Then

$$U^{HL}_1 = U^{HL}_l(G^H_1, 0) > U^{HL}_l(G^L_1, 0) = U^{HL}_1$$

where the first inequality follows by the fact that $G^H_1$ is optimal and $G^H_1 \neq G^L_1$, and the second inequality follows from Step 1.

(case iii) Suppose that $D^{max} > 0$. Then

$$U^{HL}_1 \geq U^{HL}_l(G_1, D^{max}) > U^{HL}_l(G_1, D^{max}) = U^{HL}_1$$

where the first inequality follows from the optimality of $G^H_1$, the second inequality follows from Step 1, and the last equality follows from Proposition 2.

(Step 4) We claim that for all $(G_1, D)$, $T^{HL}(G_1, D) \leq T^k(G_1, D)$, with equality if and only if $G_1 \geq \tilde{G}_2/\delta$. The claim follows from the following 4 properties, which are easily verified:

(a) $G^H_2$ and $G^L_2$ are continuous functions;
(b) $G^H_2(G^H_1, \delta, 0) = G^L_2(G^H_1, \delta, 0)$;
(c) There exists a function $G^H_2(D)$ such that: (i) for $\delta G_1 > \hat{G}^H_2(D)$, $\partial T^H_2/\partial D = 0$ and $\partial T^H_2/\partial G_1 = \delta$; and (ii) for $\delta G_1 < \hat{G}^H_2(D)$, $\partial T^H_2/\partial G_1 > 0$ and $\partial T^H_2/\partial D < 0$;
(d) $\partial T^H_2/\partial D = 0$, and $\partial T^H_2/\partial G_1$ equals 0 if $G_1 < \tilde{G}_2/\delta$ and $\delta$ otherwise.

(Step 5) We claim that $c^H_2(G_1, D) \leq c^L_2(G_1, D)$, with equality if and only if $D = 0$ and $G_1 \geq \tilde{G}_2/\delta$. Proposition 1 implies that $c^H_2(G_1, D) = w - \bar{p}/N$. The claim then follows from the following 3 properties which are easily verified:
(a) \( c^{HL} \) is a continuous function;
(b) \( c_2^{HL}(G, \delta, 0) = w - \bar{p}N \);
(c) There exists a function \( \hat{G}^{HL}_2(D) \) such that: (i) for \( \delta G_1 > \hat{G}^{HL}_2(D) \), \( \partial c_2\partial G_1 = 0 \) and \( \partial c_2\partial D < 0 \), and (ii) for \( \delta G_1 < \hat{G}^{HL}_2(D) \), \( \partial c_2\partial G_1 > 0 \) and \( \partial c_2\partial D < 0 \).

(Step 6) We claim that \( \delta \theta_2 < \theta_1 \) implies that \( \partial v_2^{HL}(D) \partial D^{max} < 0 \) (whenever it is differentiable). We use the following two facts. First, by the normality of preferences \( \partial G_1^{HL} / \partial D^{max} \in (0, 1) \). Second, as long as \( \phi_2 < 1 \), generation 1 always chooses the maximum level of debt. There are two cases to consider:

(case i) \( \delta G_1 < \hat{G}^{HL}_2(D) \). In this case

\[
\frac{\partial v_2^{HL}}{\partial D^{max}} = u_2' \left[ - \frac{1}{N} + \frac{\delta \theta_2}{\theta_1} \frac{\partial G_1^{HL}}{\partial D^{max}} \right]
\]

which is negative.

(case ii) \( \delta G_1 > \hat{G}^{HL}_2(D) \). In this case

\[
\frac{\partial v_2^{HL}}{\partial D^{max}} = - \frac{u_2'}{N} + \frac{\delta g'}{\theta_1} \frac{\partial G_1^{HL}}{\partial D^{max}}.
\]

This term is also negative since \( G_1 > \hat{G}^{HL}_2(D) \) implies that \( N \delta g'/u_2 < \theta_2 \).

Finally, note that the function \( V_2^{HL} \) is continuous since \( p^{HL} \) and \( G_2^{HL} \) are continuous.

(Step 7) Now consider the impact of the change on generation 2. There are three cases to consider:

(case i) Suppose that \( D^{max} = 0 \) and \( G_2^{HL}(G_1^{HL}, 0) = 0 \). By Step 2, it follows that \( G_1^{HL} = G_1^T \). Steps 4 and 5 then imply that \( \hat{U}_2^{HL} = \hat{U}_2^{TL} \).

(case ii) Suppose that \( D^{max} = 0 \) and \( G_2^{HL}(G_1^{HL}, 0) > 0 \). It then follows that \( G_1^{HL} < \hat{G}_2 \delta \). By Step 2, \( G_1^{HL} < G_1^T \). By Step 4, \( T_2^{HL}(G_1^{HL}, 0) < T_2^L(G_1^T, 0) \). By Step 5, \( c_2^{HL}(G_1^{HL}, 0) < c_2^T(G_1^T, 0) \). It follows that \( \hat{U}_2^{HL} < \hat{U}_2^{TL} \).

(case iii) Suppose that \( D^{max} > 0 \). By Proposition 2, \( V_2(D^{max}) = V_2^L(0) \). By the previous two cases, \( V_2(0) \geq V_2^{HL}(0) \). Finally, by Step 6, \( V_2^L(0) > V_2^{HL}(D^{max}) \) as long as \( \delta \theta_2 < \theta_1 \).

**PROOF OF PROPOSITION 7:**

As we have shown before, the capitalization functions for the two institutions are given by

\[
p^{HL}(G_1, D) = \pi \left( w - (1 - \phi_2) \frac{D + \theta_2 G_2^{HL}(G_1, D)}{N} \right) - \phi_2 (D + \theta_2 G_2^{HL}(G_1, D))
\]

and

\[
p^M(G_1, D) = \pi \left( w - (1 - \phi_2) \frac{D}{N} \right) - (\phi_2 D + \theta_2 G_2^M(G_1, D)).
\]
This uses the fact that $\phi_2 = \mu_2$.

(Step 1) We claim that the reform increases the welfare of the first generation. It suffices to show that

\[(A45) \quad p^H(G_1^{HL}, D^{max}) + D^{max} = p^M(G_1^{HL}, D^{max} + dD^{max}) + D^{max} + dD^{max}\]

which implies that generation 1 can achieve the same allocation after the reform simply by choosing $(G_1^{HL}, D^{max} + dD^{max})$. By revealed preference, generation 1 must therefore be better off. Straightforward algebra shows that (A45) holds as long as $G_2^{HL}(G_1^{HL}, D^{max}) = G_2^{M}(G_1^{HL}, D^{max} + dD^{max})$. As shown in (23), generation 2’s electoral problem depends only on the level of $G_1$ that it receives, and on $p^k(G_1, D) + D$. It follows that (A45) holds if and only if $G_2^{HL}(G_1^{HL}, D^{max}) = G_2^{M}(G_1^{HL}, D^{max} + dD^{max})$.

(Step 2) We claim that generation 2 is at least as well-off after the reform. Using (A44), straightforward algebra shows that the consumption of generation 2 is not affected from the reform. The FOCs of generation 2’s electoral problem then imply that their total consumption of IPGs does not decrease in equilibrium. The claim immediately follows.

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